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Choquet integral for criteria aggregation in the flexible job-shop scheduling problems

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Abstract

Most complex scheduling problems are combinatorial problems and difficult to solve. That is why, several methods focus on the optimization according to a single criterion such as makespan, workloads of machines, waiting times, etc. In this paper, the Choquet integral is introduced as a general tool for dealing with multiple criteria decision making and used in optimization flexible job-shop scheduling problems. The considered optimization problem is based of the Genetic Algorithm (GA) used as objective function the Choquet integral for criteria aggregation. Then lower bounds are defined for each criterion. Presented examples illustrate theoretical considerations and show the efficiency of the proposed approach.

Keywords: Flexible job-shop scheduling; Multi-objective optimization; Genetic algorithms; Choquet integral; Aggregation

1. Introduction

The flexible job-shop scheduling problem (FJSP) is known in the literature as one of the hardest optimization problems [2]. In lot of cases, the combination of goals and resources has an exponentially increasing search space. Approached methods are then preferred to exact methods and have given good solutions near the optimal one. The scheduling problem of a FJSP consists of a routing sub-problem, that is, assigning each operation to a machine out of a set of capable machines and the scheduling sub-problem, which consists of sequencing the assigned operations on all machines in order to obtain a feasible schedule minimizing a predefined objective function. The FJSP mainly presents two difficulties. The first one is to assign each operation to a machine, and the second one is to schedule these operations in order to make a predefined objective minimal.

In this paper, an aggregative approach is proposed for solving multi-objective optimization FJSP based on the evolutionary algorithms. This approach makes it possible to construct a set of satisfactory solutions according to the preferences of the decision-maker. The considered objective is to minimize makespan, the workload of the critical machine, the total workload of machines, the penalties of earliness/tardiness, and the production cost. Thus, in Sections 2 and 3, the ordered weighted averaging (OWA) operators and the Choquet Integral aggregative methods are defined and the proposed approach is described. The discussion about the use of the OWA operators is presented in Section 4. In Section 5, a multi-objective optimization by the genetic algorithm for solving FJSP is proposed. The two last sections are devoted to the formulation of some problems and to corresponding results.

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2. Proposed approach—basic idea

In order to be efficient, a multi-objective optimization has to give good results according to the preferred choices given by the decision-makers. It has therefore to solve the problem of the solutions evaluation and explore intelligently the research space to build feasible and satisfactory solution. To reach these objectives, the proposed method is based on the processing of the two following problems:

- *the problem of multi-criteria evaluation*: the criteria do not obviously have the same importance in the eyes of the decision-makers [7]. To solve the problems of evaluation, the aggregation operator proposed is the Choquet integral. This operator allows taking into account decision-maker preference as well as interaction between criteria, which are not possible to do with the OWA operator. Thus, the all criteria are reduced, by taking into account of their relative importances and interactions, to only one objective.
- *the problem of multi-criteria optimization*: the resolution of the problem is based on the algorithm genetic. With this intention, the objective function is based on the Choquet integral used for criteria aggregation.

3. Aggregative methods

Till the beginning of the nineties, the Sugeno integral was used as the aggregation tool for computing an average global score, taking into account the importance of criteria expressed by a fuzzy measure. Then, after use the Choquet integral [1] by Murofushi and Sugeno [11,13], which is an extension of classical Lebesgue integral, the well-known weighted sum was quickly adopted among practitioners. Later, the properties of Choquet and Sugeno integrals, as an aggregation operator, were studied in depth, and their connection with OWA operators in their usual additive form [15].

For the sake of simplicity, but without loss of generality, the numerical score of criterion C_i is a_i ranges in [0,1].

3.1. Ordered weighted averaging operators

The operator of aggregation C_{OWA}, introduced by Yager [15], is expressed by:

$$C_{\mathrm{OWA}}(x) = \sum_{i=1}^{n_c} w_i a_i \tag{1}$$

where $w = (w_1, w_2, \ldots, w_{n_c})$ is a weight vector such that $w_i \in [0, 1]$, $\sum_{i=1}^{n_c} w_i = 1$, and a_i the value of criteria C_i.

The OWA operators allow within the framework of the aggregation multi-criterion to represent mainly the following two concepts, difficult to consider simultaneously [9]:

- the decision-maker can act on the aggregation by imposing a more or less strict satisfaction of the criteria set: one can represent all the going nuances of "all the criteria must be satisfied at least a criterion has to be satisfied", intuitively due to the connection with linguistic quantifications,
- the introduction of the notion of level-headedness between criteria is possible: the more a criterion is important in the final decision-taking, the more its weight is high.

3.2. Basic material on fuzzy measures and Choquet integral

In order to be self-contained as far as possible, are given, in this section, necessary definitions, adapted for multi-criteria decision making.

Let consider a finite universal set $N_c = \{1, \ldots, n_c\}$, which can be thought as an index set of the given criteria [4].

Definition 3.1. A fuzzy measure over $N_c = \{1, \ldots, n_c\}$ is a set function μ: $P(N_c) \to [0, 1]$, such that:

1. $\mu(\phi)$=0, $\mu(N_c) = 1$
2. $\mu(A) \le \mu(B)$ whenever $A \subset B \subset N_c$