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The effects of emission trading on production and inventories in the Arrow–Karlin model

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Abstract

The paper deals with the effect of introduction of tradable permits on the production–inventory strategy of a firm. It is assumed that the firm will minimize its costs. The cost function consists of linear holding and convex production costs. After introducing emission trading, the cost function will contain a linear emission procurement/selling cost. We will compare the optimal production–inventory strategy before tradable permits and thereafter. The mathematical investigation is based on the well-known dynamic Arrow–Karlin model.

Keywords: Emission trading; Tradable permits; Environmental licences; Optimal control; Production; Inventory; Environmental management

1. Introduction

The aim of the paper is to analyze the effects of tradable permits (emission trading or environmental licence) on production and inventory. Tradable permits are a combined instrument of the environmental regulation (Field, 1997). The government can control emission of the firms with pollution (emission) taxes and/or environmental standards. These two instruments of environmental regulation are not market conforming in the sense that it is the government that influences the functioning of the firm and not the market. In the case of tradable permits, the firm can purchase or sell his pollution level on the market of pollution rights. The price of the emissions (pollutions) is determined on the emission market, so it is market conforming. The purchased/sold pollution level acts for the firm, on one hand, as an environmental standard (upper bound) and on the other hand, as a pollution tax (extra costs or revenue). We will investigate the effect of the introduction of emission trading for a production firm.

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The basis of the investigations is the well-known Arrow–Karlin-type dynamic production–inventory model (Arrow and Karlin, 1958). In this model, the inventory holding costs are linear, the production costs are non-decreasing and convex function of the production level. We will assume that the basis of the initial emission rights allocation is the pollution under the minimal cost production–inventory strategy before introducing the emission trading program. The firm is able to purchase its rights or sell them after the introduction. It means that the basic Arrow–Karlin model will be expanded with a pollution constraint, where the pollution constraint can be modified with the purchased/sold emission rights. We will investigate the instantaneous effects of the introduction of emission trading. It is also assumed that the permits are totally divisible and the costs of the purchased/sold licences are linear, i.e. the price of the emission rights is constant in the planning horizon. This assumption about the price of tradable permits can be held, because the planning horizon is relatively short in this examination and the prices do not change in a short-time interval. The instantaneous pollution is also a non-decreasing and convex function of the production level. The goal of the decision maker is to minimize the relevant costs after the installation process. It is asked: how much pollution rights to procure or sell and what are the effects of these rights on the production–inventory strategy.

The paper is organized as follows. In Section 2 the variables and parameters will be defined and two models will be shown to be compared: the basic Arrow–Karlin model and the model modified with pollution. Section 3 describes some properties of an optimal solution compared to the basic model. With the help of these qualitative characteristics of the model, it can be shown that the investigated model can be transformed in the model with pollution tax examined by Wirl (1991). The optimal solution is presented in Section 4. We will show that emission trading smoothes the production level of the firm and the inventory level will be higher. In Section 5 we illustrate the results of the paper with a numerical example. In the last section we will summarize the results of the paper.

2. The model with tradable permits

Before presenting the models, we summarize the notations of the model:

Decision variables:

$I(t)$	inventory status in point t, non-negative
$P(t)$	production rate at time t, non-negative

The control variable of the models is the production rate $P(t)$ and the state variable is the inventory level $I(t)$.

Parameters:

T	length of the planning horizon, positive
I_0	the initial inventory level at the beginning of the planning horizon, non-negative
$S(t)$	the known determinative demand rate at time t, non-negative
h	the linear inventory holding costs, positive
$F(P(t))$	the costs of production at time t, non-decreasing, strictly convex function, non-negative
d	the linear costs of one unit tradable permit, positive
$g(P(t))$	the rate of emission at time t, non-decreasing, strictly convex function, non-negative
α	the cumulated emission in the planning horizon, $\alpha = \int_0^T g(P(t))\,\mathrm{d}t$
$\Delta\alpha$	the sold/purchased tradable permit for the planning horizon, negative in case of sale and positive in case of purchase

The material and cost flow of the model is depicted in Fig. 1.