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Capacitated lot sizing with linked lots for general product structures in job shops ☆

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ABSTRACT

In this paper, we propose a mixed integer programming (MIP) model for a multi-level multi resource capacitated lot sizing and scheduling problem with a set of constraints to track dependent demand balances, that is, the amount left over after allocating the available inventory to the dependent demands. A part of this leftover amount may be kept as a reservation quantity to meet dependent demands of the following period under capacity restrictions. These constraints are necessary because we assume independent demands as well as dependent demands for all items in the product structure. They also are used to tighten the domain of on hand and backorder inventory levels. Although we allow backorders for independent demands only, this is not possible for dependent demands as backorders will disturb the whole demand balance of the product structure. Determination of setup costs is a crucial task when developing lot sizing and scheduling models, especially in a capacitated manufacturing environment with backorders. In this respect, the capacitated lot sizing with linked lot sizes (CLSPL) model we formulate needs not to consider setup costs to avoid unnecessary setups thanks to the new set of constraints, and to obtain feasible lot sizes and schedules. Finally, a numerical example and computational results in a job shop environment are also given, and future research directions are provided.

1. Background and motivation

Material Requirements Planning (MRP), developed by Orlicky (1976), is the most popular production planning and scheduling system in practice. MRP provides the right part at the right time for the right customer, i.e., it aims to plan the end item requirements of the master production schedule.

First, MRP systems are characterized by their rapid adaptability to dynamic changes in a production/inventory system, and ability to determine the production and inventory requirement several periods in advance. Limitations, for example, are its inability to perform comprehensive capacity planning, using constant and inflated lead times, and its lack of a fluent shop floor extension due to myopic solution methodology (see Pochet & Woolsey, 2006). To address these limitations, it may be necessary to develop an optimization approach to reach the desired goal of simultaneously improving the productivity and flexibility of an MRP system. A comprehensive survey of the different optimization approaches, that were developed in the lot sizing and scheduling literature can be found in Drexl and Kimms (1997).

Lot sizing and scheduling literature generally is categorized into two groups, which are the small and big time buckets. Grouping is generally based on specifications of production and the length of planning period. While at most two different items can be produced on a resource in small time bucket models, number of different lots that can be produced is not restricted in big-bucket models. Since at most one setup is allowed in small time bucket models, the sequence of different lots is already known. In other words, small time bucket models solve the lot sizing and scheduling problem together. On the other hand, a primitive form of big time bucket models, which is known as capacitated lot sizing problem (CLSP), cannot answer the scheduling question. However, recent studies on big time buckets try to determine at least the first and the last lots by linking adjacent periods.

While periods in small bucket models usually correspond to small time slots such as hours or shifts, big-bucket models deal with planning horizons usually less than 6 months i.e., days, weeks or months (Drexl & Kimms, 1997). The main characteristic of big-bucket models is that they do not restrict the number of items produced in any period.

The capacitated lot sizing problem (CLSP) is typical of big-bucket models. It is an extension of the Wagner–Whitin model to integrate capacity limitations. Since the size of a bucket is larger than the setup times, the error due to nonpreservation of the setup state between adjacent periods is negligible. Different forms of the mixed integer programming model for solving the CLSP are surveyed in Karimi, Ghomi, and Wilson (2003).

In a manufacturing environment, there are instances where developing a feasible schedule is only possible when setup states

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are carried over from one period to another. Setup carryover is the continuation of a production run from one period to the next without an additional setup. Setup carryover in big-bucket models is concerned with "partial sequencing" of items. The sequence of items scheduled between the first and last does not affect the total required setup time unless the items are sequence dependent, where setups can be done in any order.

The complexity of modeling setup carryover in CLSP problems is why it has not received much attention in the literature. Gopalakrishnan, Miller, and Schmidt (1995) present a formulation of the CLSP with setup carryover, which is not effective because of its extensive use of binary variables. In addition, their model assumes all items have identical setup cost and times. Gopalakrishnan (2000) later improves his model to incorporate item dependent setup times and costs. Haase's (1998) CLSP problem restricts the carryover to at most one period. Sox and Gao (1999) consider a capacitated lot sizing problem with linked lot sizes (CLSPL) but without setup times. Their model formulation is based on the shortest route representation of the problem. They also indicate that restricting the carrying of setup states over both periods causes a maximum deviation from optimality of 2.19%; however, a solution was reached faster. Suerie and Stadtler (2003), present a new MIP model with valid inequalities to yield a tight formulation of CLSPL that this paper is partially based on. While they prove the infeasibility of carrying exactly one setup state from one period to another for some cases, their model formulation ignores the necessity of carrying setup state. In other words, whether the same item continues to be produced or not, setup state of a resource has to be always carried. Tempelmeier and Buschkühl (2008) develop a Lagrangean heuristic for CLPSL problem, that assumes no backorders and unit lead times. Sahling, Buschkuhl, Tempelmeier, and Helber (2009) improve the work of Tempelmeier and Buschkühl (2008) by incorporating multi-period setup carryover, and propose a fix and optimize heuristic. However, their problem formulation ignores backlogging and includes setup minimization for consistent plans. In addition, Sahling et al. (2009) assume infinite capacity of overtime while they try to minimize it using fix and optimize heuristic. Besides, they ignore the possibility of a setup occurrence when a setup state of one item is carried over multiperiods. Akartunali and Miller (2009) propose a heuristic framework that can generate high quality feasible solutions for big bucket multi-level production planning problems without setup carryovers. However, their assumptions ignore external demand case for sub-level items, hence allow backlogging for only end items. Meanwhile, their model formulation also needs setup minimization to get consistent plans (see Appendix B for definition of consistency).

All of the lot sizing and scheduling models mentioned above strongly need to minimize the number of setups to obtain consistent plans. In this paper we formulate a new CLSPL model that needs not to minimize setups for consistency. By consistency we mean avoiding unnecessary setups (see Appendix B).

Using lot sizing and scheduling models in master production schedules has received less attention in the literature (Askin & Goldberg, 2002; Voß & Woodruf, 2003). Even though the available studies allow backordering for the independent part of total demand for all items, performance of constraints that they use to protect dependent demand consistency do not work well in level by level decomposition approaches (see for instance Nagendra & Das, 2001; Ornek & Cengiz, 2006). The model developed in this paper includes tighter constraints to restrict the domains of backordering and holding inventories (see Appendix A).

Finally, we describe the problem environment that is covered in this paper and list the contributions as follows.

The model formulation presented in this paper includes the case of selling the individual component items, which is the case of independent demands exist. Consequently, our model allows backlogging for all items in the bill of material for independent demands only. We further develop a new set of constraints to assure dependent demand consistency when backlogging is allowed. These constraints guarantee a stock of units to meet dependent demands of the following periods under capacity restrictions (see Appendix A).

The model formulation presented here could be applied to all types of product structures, where a product flow is assumed to follow a general job shop. In more detail, components at different levels in the product structure have no dedicated resources. In other words, a resource can be shared by components at different levels, i.e., an item can visit a resource more than once as a component of other items. Of course our formulation can be used in other types of product flows by just rearranging the routing parameters.

Also the model developed in this paper can easily be extended to incorporate other types of supply replenishments, i.e. purchasing, and outsourcing.

The scheduling logic of the model here in is a modified version of the CLSPL presented by Suerie and Stadtler (2003), where we extend their setup carryover formulation to eliminate its drawbacks and to enforce the memory of the model. Here the word "memory" is used in the sense that the model remembers the setup state in a period and carries it to the following period (see Appendix B). In addition, to our knowledge, the model formulation presented in this paper is the first study on application of setup carryover with backlogging allowed.

The rest of the paper is organized as follows. We provide a detailed formulation of the model in the next section, which is followed by a numerical example and computational results in Section 3. Summary, conclusions and future research directions are presented in Section 4. Justification for some of the constraints and comparisons with specific models in the literature are given in the Appendices A and B.

2. Model development

Notation, indices, parameters and decision variables that are used in the rest of this paper is as follows:

Indices and sets

$i, j = 1, \ldots, V$ index of end items,
$i, j = V{+}1, \ldots, N$ index of component items,
$k = 1, \ldots, K$ index of resources,
$t = 1, \ldots, T$ index of time periods,
A_i set of direct successors of component item i,
S_k set of items that can be manufactured on resource k,
M_i set of resources which item i can be manufactured,

Parameters

$c^{(b)}(i)$ unit backorder cost for item i,
$c^{(h)}(i)$ unit holding cost for item i,
$l(i)$ manufacturing lead time for item i,
$e(i, j)$ number of units of i required to produce one unit of j ($j \in A_i$) (gozinto coefficient),
$d(i, t)$ independent demand for item i from outside sources at period t,
$I^{+}(i, 0)$ initial on hand inventory quantity for item i,
$I^{-}(i, 0)$ initial backorder quantity for item i,
$p(i, k)$ time to process one unit of item i on resource k,
$s(i, k)$ setup time of item i on resource k,
$c(k, t)$ capacity of resource k in period t,
$b(i)$ minimum quantity for item i that can be manufactured physically in terms of stock keeping unit of item i,
$m(i, k)$ big number for item i on resource k and equals to the maximum production quantity,
$L(i, k, 0)$ initial setup state for item i on resource k (0 or 1).