Transportation Research Part E 48 (2012) 937–957

Contents lists available at SciVerse ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

# Optimal distance tolls under congestion pricing and continuously distributed value of time

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ARTICLE INFO

*Article history:*
Received 12 August 2011
Received in revised form 13 February 2012
Accepted 27 March 2012

*Keywords:*
Distance-based toll
Cordon-based congestion pricing
Stochastic user equilibrium
Continuously distributed value-of-time
Mathematical programming with equilibrium constraints
Genetic algorithm

ABSTRACT

This paper addresses the optimal distance-based toll design problem for cordon-based congestion pricing schemes. The optimal distance tolls are determined by a positive and non-decreasing toll-charge function with respect to the travel distance. Each feasible toll-charge function is evaluated by a probit-based SUE (Stochastic User Equilibrium) problem with elastic demand, asymmetric link travel time functions, and continuously distributed VOT, solved by a convergent Cost Averaging (CA) method. The toll design problem is formulated as a mixed-integer mathematical programming with equilibrium constraints (MPEC) model, which is solved by a Hybrid GA (Genetic Algorithm)–CA method. Finally, the proposed models and algorithms are assessed by two numerical examples.



## 1. Introduction

Acting as an economic lever for traffic demand management in urban metropolises, congestion pricing has received a lot of attention both academically and practically. Following two pioneer practices, Singapore in 1975 (Phang and Toh, 1997; Li, 1999) and Norwegian cities in the mid 1980s (Langmyhr, 2001), implementations of congestion pricing have been accelerated when it comes to the new century, for instance, London in 2003 (Santos, 2008) and Stockholm in 2006 (Eliasson, 2009). Nearly all these congestion pricing practices adopt the cordon-based congestion pricing scheme: certain district in urban area is encircled by a pricing cordon and any vehicle passing through the cordon is charged. By affecting drivers' route choice plans and subsequently restricting the total number of vehicles entering the encircled district, the cordon-based congestion pricing scheme is taken as an effective tool to mitigate traffic congestion, and it is also convenient for practical operations (May et al., 2002; Akiyama and Okushima, 2006).

All the implemented cordon-based congestion pricing schemes currently use a flat toll-charge method including the daily licensing basis charge (Santos, 2008) and the pay-per-entry basis charge, regardless of the travel distance or time in the pricing cordons. This flat toll-charge method, however, is inequitable because it undercharges long journeys and over-restrains short ones (May et al., 2008). In addition, the flat toll-charge is also not fully efficient for congestion mitigation, since some drivers may intentionally use more road segments in the cordon area, in order to maximize the utility of their investments. To cope with these drawbacks of flat toll-charge method, May and Milne (2000) examined three possible alternative toll-charge methods: (a) *time-based* method according to the time consumed in traversing a cordon; (b) *congestion-based* method in accordance with the travel time spent in congestion; and (c) *distance-based* method relied on the distance travelled. May

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1366-5545/$ - see front matter 
http://dx.doi.org/10.1016/j.tre.2012.04.004

and Milne (2000) concluded that these three methods outperformed the flat toll-charge method in terms of traffic congestion mitigation. The first two toll-charge methods, to some extent, encourage aggressive driving behaviors and may cause more traffic safety issues. They are hence not adopted in practical trials (Richards et al., 1996). The distance-based toll-charge method can be efficiently implemented with the aid of the global positioning system (GPS) and an in-vehicle unit integrating a GPS receiver, a digital map and a general packet radio service (GPRS) communication device. It is more preferable for the next generation of congestion pricing schemes. Note that toll charges for the distance-based toll-charge method should be a function of the travel distance in each pricing cordon, which is termed distance-based toll-charge function. It thus makes the toll charges, in most cases, non-additive (Bekhor and Toledo, 2005), i.e., the overall toll charge on an itinerary/path cannot be proportionally divided to be toll charges on its component links.

Land Transport Authority (LTA) of Singapore has updated the bus fares for public transport system to be distance-based. The bus fares are determined by a universal fare structure table, and as shown by Fig. 1 the bus fare in such case is a non-linear function of travel distance. The distance-based bus fare has eliminated the transfer fare penalty, thus it can encourage the use of public transport system. Meanwhile, LTA intends to convert its Electronic Road Pricing (ERP) system from the current pay-per-entry charge to the distance-based charge termed as the second generation ERP system (Ohno et al., 2007). Similar to the distance-based bus fare indicated in Fig. 1, a distance-based toll charge function is needed for second generation ERP system. It is essential to determine a proper toll-charge function that is beneficial for the whole road network. Therefore, it is a new research issue with practical importance to estimate a distance-based toll-charge function that maximizes the total social benefit (TSB). As a side note, we point out that apart from the cordon-based pricing schemes Singapore's ERP system also has link-based tolls on partial expressways and arterial roads. Yet, the link-based tolls are not taken into consideration in this paper.

Analysis of the optimal distance-based toll design problem has to take into account the behavior of drivers in their route (or path) choice, which is assumed to obey the probit-based stochastic user equilibrium (SUE) principle, namely, drivers are assumed to have a normally distributed perception error on the actual path travel time. Other than the actual travel time and perception error, overall travel impedance on each path (termed as generalized path travel time) also includes a toll charge. The toll charge should be converted into time-units using the drivers' value of time (VOT). It is well-known that the VOT varies among different drivers due to their different levels of income and trip emergency. To reflect this variation, it is more rational to formulate the VOT as a continuously distributed random variable rather than a fixed (mean) value (Verhoef and Small, 2004; Small et al., 2005; van den Berg and Verhoef, 2011). The continuously distributed VOT results in another random term in the generalized path travel time, besides the perception error. A computational model is necessitated to describe the probit-based SUE principle with the continuously distributed VOT due to the inapplicability of the existing models developed for the SUE problems. Assuming the non-additive distance-based toll charges as well as a continuously distributed VOT, this paper aims to investigate the proposed optimal distance-based toll design problem in model development and algorithm design.

### 1.1. Relevant studies

There are numerous achievements on the congestion pricing studies (Small, 1992; Yang and Huang, 2005; Lawphongpanich et al., 2006; Han and Yang, 2008; Verhoef et al., 2008; etc.). Among these studies, the cordon-based congestion pricing scheme is usually regarded as a particular second-best pricing solution (McDonald, 1995; Verhoef et al., 1996; Verhoef, 2002; Yang and Zhang, 2002; Meng et al., 2005; Meng and Liu, 2011a, 2012). Based on given cordon locations, optimal toll design problem for cordon-based congestion pricing aims to identify a proper toll fare solution that optimizes a system-wide objective function, such as the TSB.

When assuming the flat toll-charge method, the optimal toll design problem for cordon-based congestion pricing schemes can be formulated as a mathematical program with equilibrium constraints (MPEC) or bi-level programming model

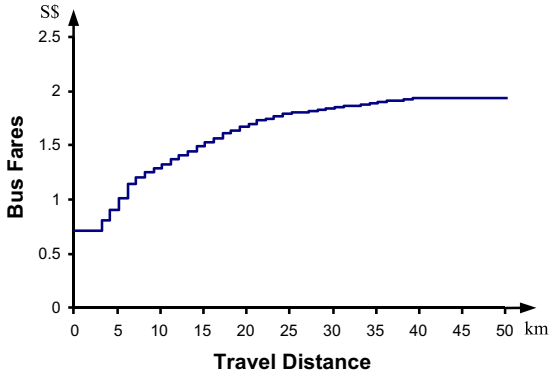


**Fig. 1.** Bus fare structure for public transport system in Singapore.