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A flexible model for the pricing of perishable assets

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ABSTRACT

We present a flexible and versatile model which addresses the problem of assigning optimal prices to assets whose value becomes zero after a fixed expiry date. (Such assets include the important example of seats on airline flights.) Our model is broad in scope, in particular encompassing the ability to deal with arrivals of customers in groups. It is highly adaptable and can be adjusted to deal with a very extensive set of circumstances.

Our approach to the problem is based on elementary and intuitively appealing ideas. We model the arrival of customers (or groups of customers) according to an inhomogeneous Poisson process. We incorporate into the model time dependent price sensitivity (which may also be described as ''time dependent elasticity of demand''). In this setting the solution to the asset pricing problem is achieved by setting up coupled systems of differential equations which are readily amenable to numerical solution via (for instance) a vectorised Runge–Kutta procedure. An attractive feature of our approach is that it unifies the treatment of discrete and continuous prices for the assets.

1. Introduction

In a large class of optimization problems (termed ''asset selling'' or ''asset pricing'' problems) introduced by Karlin [1], a vendor has a stock of a finite number of items which must be sold by a given deadline, otherwise their value becomes zero. (Examples of such assets include seats on a given airline flight, rooms in a hotel for a given night, and advertising slots on a particular television program.) In Karlin's version of the problem, customers arrive according to a given process and make offers to buy a certain number of items. The analyst's objective is to determine if the current offer is acceptable or not.

Since Karlin's initial work many versions and extension of this idea have been studied, in particular including applications to the problem of pricing airfares. See [2–7].

In reality, especially in the airline context, it is rare for customers to make offers to the vendor. More usually an arriving customer, instead of making an offer, is quoted a price by the vendor. The probability that the customer will choose to purchase the asset in question depends upon the quoted price. The objective of the vendor is to choose a time and stock-level dependent pricing policy so as to maximize expected revenue. Reformulating the problem in this way alters the form of its solution remarkably. It is this form of the problem which is addressed in this paper.

Recent important work on this more realistic version of the problem includes that of Feng and Xiao [8,9], and of Zhao and Zheng [10].

The ideas discussed in this paper may be considered to constitute a part of the more general concept of optimal pricing and revenue management, which forms an active area of on-going research. See for instance [11,12].

Related ideas on perishable asset revenue management may be found in [13]. Related ideas are also to be found in the context of the ''news vendor'' or ''news boy'' problem, in settings in which the vendor is allowed to vary the price at which items are sold. This problem continues to generate a great deal of active research. See for instance [14–20].

A very different approach to the dynamic pricing of assets may be found in [21]. Discussions of perishable asset pricing in the context of the hotel industry may be found in [22,23]. Other related ideas may be found in [24,25].

A thorough treatment of the whole area of revenue management is to be found in [26]. A useful summary of the area, including the ideas of dynamic pricing is provided by [27]. A model which is fairly closely related to ours is examined in [28]. This model however effectively combines the ideas of a random process of customer arrivals and of random acceptance of offers by prospective customers, into a single ''realized demand process''. This model requires the existence of a ''null price'' (which pertains when the stock of items is exhausted). Our model has no such requirement.

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Our model is expressed in terms of individual rather than aggregate demand. This and the fact that we decouple customer arrivals and the acceptance process, making the latter random, avoid the difficulty of dealing with the distinction between "myopic" and "strategic" customers (see [26, p. 182]). In our treatment this distinction is subsumed in the randomness of a customer's accepting a given offer. This seems reasonable: the nature of a customer (myopic or strategic) is unpredictable and hence random from the point of view of the vendor.

None of the existing literature, as far as we have been able to discover, deals with the issue of customers arriving in groups. Our model is capable of handling this phenomenon.

2. The basic model assumptions

We preface this section by remarking that throughout the forthcoming development the symbol t will represent *residual* time, that is the time *remaining* until the deadline (the time at which the assets cease to have any value, e.g. the departure time of an airline flight).

We shall now set out a precise description of the model which we are about to develop and analyze, and of the scenario to which it applies. This description is encapsulated in the following assumptions:

A.1 A vendor has a stock of items which must be sold by residual time 0. The vendor's objective is to determine an optimal pricing policy, that is, one which maximizes the expected revenue.

A.2 Customers arrive at the sales point in groups of random size J, the probability distribution of J being given by $\Pr(J=j)=\pi(j)$, $j=1,2,\ldots$, with all $\pi(j)\geq 0$ and $\sum_j \pi(j)=1$.

A.3 The arrival times of the *groups* are random and follow an inhomogeneous Poisson process with intensity $\lambda(t)$.

A.4 On arrival a group is quoted a (single) price for the items on sale. In deciding on a price to quote, the vendor will take into account the time, the size q of the stock remaining, and possibly the group size. The price quoted (at residual time t, with a stock of size q remaining) is denoted by $x_q(t)$ if group size is not taken into account, and by $x_{qj}(t)$ (where j is the group size) if group size *is* taken into account.

A.5 If the size j of the arriving group is less than or equal to the current stock size q then the group collectively either accepts the quote and purchases j items, or rejects it and makes no purchase. The probability that the group accepts a quote of price x per item is given by a *price sensitivity function* $S_j(x,t)$.

A.6 If the size j of the arriving group is greater than the current stock size q, then the group will nevertheless *consider*, with probability α, making the purchase of (all of) the remaining q items. With probability $1-\alpha$ the group will decline to consider any offer.

A.7 If the size j of the arriving group is greater than the current stock size q, and if the group does decide to consider the offer, then the group is treated as if it were a group of size q. That is, the price offered (where group size is taken into account) is the price that would be offered to a group of size q. Also it is assumed that the appropriate price sensitivity function is $S_q(x,t)$.

The variables and $x_q(t)$ or $x_{qj}(t)$ are decision variables. We will formulate systems of coupled differential equations to evaluate them. Our procedure is related to but different from the backward recursion approach of dynamic programming.

We remark that Assumption A.3 is commonly used in studies which deal with arrivals that are stochastic in nature. The paper by McGill and van Ryzin [27] lists several such studies including [29,28,30,10]. With respect to Assumption A.5 we remark that (in the context of airline seat pricing) business travelers are less sensitive to prices than other travelers such as tourists. Business travelers are constrained, more often than other types of traveler, to travel "at the last minute", whence it makes sense to allow the dependence of the price sensitivity function upon time as well as price. The dependence of the price sensitivity function upon group size makes the model more flexible and more realistic.

The group size distribution function $\pi(j)$, the arrival intensity function $\lambda(t)$, the probability α and the price sensitivity functions $S_j(x,t)$ are all assumed to be known to the vendor. The vendor of course also knows the current values of t and q.

In the simplest instance customers always arrive singly (i.e. all groups are of size 1, or in other words π is the degenerate distribution $\pi(1)=1$, $\pi(j)=0$ for $j>1$). In such an instance there would be only one price sensitivity function $S(x,t)=S_1(x,t)$. As far as we are aware this is the only type of random arrival of customers which has previously been considered in any way in the literature.

Various assumptions could be made about the manner in which groups reach a consensus as to whether to accept a given offer. Such assumptions would determine the nature of the price sensitivity functions $S_j(x,t)$, One possible class of assumptions consists of those in which a group consensus is determined on the basis of individual decisions made *independently* by each of the members of the group. The decisions of the individual group members would be made according to a single price sensitivity function $S(x,t)$. In these circumstances $S_j(x,t)$ is given by an expression involving $S(x,t)$ and j.

It could for instance be assumed that the group will accept the offer only if all members of the group accept it. Under this assumption $S_j(x,t)=S(x,t)^j$. Other possibilities would be to assume that the group will accept the offer if any *one* of the members accepts it, or if a majority (strict or otherwise) of the members accepts it. The expressions for $S_j(x,t)$ in terms of $S(x,t)$ and j become more complex in the context of these other possibilities.

3. Pricing policies and expected values of stocks

We define a "pricing policy" $\boldsymbol{x}$ specifying the prices to be asked for an item at time t (we remind the reader that t denotes residual time) if there are q items in stock at that time. In the setting in which prices are independent of the size of the arriving group of customers this will consist of a set of functions $\boldsymbol{x}=\{x_q(t): q=1,\ldots,Q\}$ where Q is the number of items in stock at the start of the process (at residual time T). In what follows we will, for brevity, refer to this setting as the "s.i.p." (singly indexed price) setting. If prices depend on group size as well as stock size the functions will be doubly indexed, i.e. we have $\boldsymbol{x}=\{x_{qj}(t): q=1,\ldots,Q, j=1,\ldots,q\}$. We will refer to this as the "d.i.p." (doubly indexed price) setting.

The price functions $x_q(t)$ or $x_{qj}(t)$ will be assumed to be at worst piecewise continuous (i.e. to have at most a finite number of jump discontinuities in the time interval under consideration). Since we may alter the price at a finite number of points without changing the expected revenue from the stock, we may also assume, without loss of generality, that these functions are right continuous.

We define $v_q(t|\boldsymbol{x})$ to be the expected value of (expected revenue from) a stock of q items at time t under pricing policy $\boldsymbol{x}$. The fundamental objective is to choose $\boldsymbol{x}$ so as to maximize $v_q(t|\boldsymbol{x})$ for all q. This is accomplished in two stages. We first establish a system of differential equations that can be solved for the $v_q(t|\boldsymbol{x})$ given a pricing policy $\boldsymbol{x}$. After having done so we derive from this system another such system which the optimal entries