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Diagnosing affine models of options pricing: Evidence from VIX ☆

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ABSTRACT

Affine jump-diffusion models have been the mainstream in options pricing because of their analytical tractability. Popular affine jump-diffusion models, however, are still unsatisfactory in describing the options data and the problem is often attributed to the diffusion term of the unobserved state variables. Using prices of variance-swaps (i.e., squared VIX) implied from options prices, we provide fresh evidence regarding the misspecification of affine jump-diffusion models, as variance-swap prices are affine functions of the state variables in a broader class of models that do not restrict the diffusion term of the state variables. We apply the nonparametric methodology used by Aït-Sahalia (1996b), supplemented with bootstrap tests and other parametric tests, to the S&P 500 index options data from January 1996 to September 2008. We find that, while the affine diffusion term of the state variables may contribute to the misspecification as the literature has suggested, the affine drift of the state variables, jump intensities, and risk premiums are also sources of misspecification.

1. Introduction

Recent advances in modeling options prices are aimed at solving the problems of volatility smile and smirk, which refer to the phenomena that the implied volatility from the Black and Scholes (1973) formula is a smile-shaped function of the strike price before the 1987 stock market crash and a decreasing function after the crash. Since the problems stem from the constant volatility assumption of the Black–Scholes model, major advances are made along the line of stochastic volatility models, in which unobserved instantaneous volatility of the underlying security follows another stochastic process and serves as an additional state variable in pricing options. More recent developments add jump components to the processes of both the price of the underlying security and the state variables which are components of stochastic volatilities. The affine jump-diffusion models make headway towards resolving the pricing issue. Important milestones include the models of Heston (1993) and Duffie, Pan, and Singleton (2000).[1] However, empirical studies show that the existing models are still inadequate in fitting the observed options prices in cross-sections too various degrees. Bakshi, Cao, and Chen (1997) find that the Heston model requires highly implausible parameters of the volatility–return correlation and the volatility-of-volatility.

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[1] Empirical analysis of equity index options using affine jump-diffusion models includes Bakshi, Cao, and Chen (1997), Bates (2000), Chernov and Ghysels (2000), Pan (2002), Eraker, Johannes, and Polson (2003), Eraker (2004), Broadie, Chernov, and Johannes (2007), among others. For recent surveys on the options pricing literature, see Broadie and Detemple (2004) and Garcia, Ghysels, and Renault (2010).

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Bates (2000) extends the stochastic volatility/jump-diffusion model to a two-factor specification with time-varying jump intensity. Pan (2002) and Eraker (2004) report improvements of models with jumps in both the underlying price and the stochastic volatility processes in the time-series dimension. The fit of cross-sections of options data, however, is still unsatisfactory even with added jump components.

The existing evidence in the literature suggests that the rejection of affine jump-diffusion models of options pricing is due to the misspecification of the diffusion term of stochastic volatility. For example, Duffie et al. (2000) suggest that the deficiency of certain specific affine jump-diffusion options pricing models is that these models unnecessarily restrict the correlation between the state variables driving the underlying returns and the stochastic volatility. Jones (2003) finds that the square-root stochastic volatility model of Heston's type is incapable of generating realistic return behavior and concludes that the stochastic volatility models in the constant elasticity-of-variance class or with a time-varying leverage effect are more consistent with the underlying asset and options data. Christoffersen, Jacobs, and Mimouni (2010) find that a stochastic volatility model with a linear diffusion term fits options prices better than the square-root process, which implies that the conditional variance of the stochastic volatility is better modeled as a quadratic function of the state variables. To sum up, some authors identify the problem with specific affine models as the restrictiveness of the diffusion term of the state variables which can be solved by using less restrictive diffusion terms within the affine class of models, while others find that the entire affine class is inadequate because of the empirical evidence of the non-affine diffusion term of the state variables.

In this paper we address the following questions. Are diffusion terms of the state variables in specific affine models the only source of the problems in options pricing? Can the problem of specific models be resolved within the class of affine jump-diffusion models by having a more flexible diffusion term? The analysis we conduct in this paper is based on one observation that, in a much wider class of models than the affine jump-diffusion models, variance-swap prices are affine functions of state variables and inherit the properties of state variables. The wider class with this property, named as the semi-affine models in this paper, is the class that imposes no restriction on the diffusion term of the state variables. We examine the affine properties of the conditional mean and conditional variance of the variance-swap prices implied from options and reject them. Since the affine property of the conditional mean of variance-swap prices does not rely on the affine property of the diffusion term of the state variables, the rejection of the affine property of the conditional mean of variance-swap prices can be traced to the inappropriate affine specifications of either the conditional mean of state variables, jump intensities, or risk premiums.

The methodology we use in this paper begins with the nonparametric method used by Aït-Sahalia (1996b) and Stanton (1997) on short-term interest rate, followed by bootstrap tests and some parametric tests. The use of nonparametric methods allows us to address issues with general affine models, rather than specific affine models. We apply our methods to the S&P 500 index options from January 1996 to September 2008. The nonparametric estimation and testing results in this paper show that the conditional mean, conditional variance, and conditional covariance of the variance-swap prices of the S&P 500 index exhibit strong non-affine properties. More specifically, the mean reversion of variance-swap prices is much faster and the volatility of variance-swap prices is much greater at the high levels of variance-swap prices than what affine functions imply. Parametric tests further confirm the results. Both nonparametric and parametric results suggest that the specifications of the affine drift of state variables, jump intensities, and risk premiums are all potential reasons for the rejection of affine jump-diffusion models. The problems of specific affine models cannot be resolved by more flexible diffusion terms not only within the affine class of models, but also within the semi-affine class of models. It should be noted, however, that our empirical evidence presented in this paper is limited to the case of S&P 500 index options, and does not necessarily extend to other data.

Our specification analysis of the affine jump-diffusion models is based on the dynamic features of the variance-swap prices inferred from cross-sections of option prices. This is in contrast to the approach in the existing literature where option pricing models are fitted to the prices of individual options. Our approach has the following advantages. First, since state variables are unobserved, complicated econometric methods have been used in the literature to estimate the models, which make it difficult to identify which aspect of the affine jump-diffusion models causes problems. This is especially so when Heston's univariate model is extended to multivariate models. Our approach of examining the conditional mean and variance of transformed state variables in parametric and nonparametric analyses is straightforward to implement and avoids complicated econometric procedures. Second, the prices of certain long-maturity, deep in- or out-of-the-money options may contain errors due to liquidity reasons. The existence of these errors makes it difficult to know whether a model is rejected because of model misspecification or because of data errors. Our approach of using variance-swap prices avoids this problem because prices of individual options are aggregated, so the impact of idiosyncratic errors is substantially reduced.

Our intended contribution of this paper is to provide evidence at a fairly general level that the misspecification of the affine models goes beyond the diffusion term of the state variables, as the literature has been focused on. Since models outside the affine class are difficult to solve, such information can be valuable to theoretical modelers in directing their efforts towards finding better models. While our empirical results are limited to the S&P 500 index options only, the methodology can be used in other cases in which sufficient cross-sections of options data are available.

The rest of the paper is organized as follows. Section 2 presents the jump-diffusion models that we investigate