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Fuzzy hierarchical TOPSIS for supplier selection

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ABSTRACT

This study simplifies the complicated metric distance method [L.S. Chen, C.H. Cheng, Selecting IS personnel using ranking fuzzy number by metric distance method, Eur. J. Operational Res. 160 (3) 2005 803–820], and proposes an algorithm to modify Chen's Fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [C.T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets Syst., 114 (2000) 1–9]. From experimental verification, Chen directly assigned the fuzzy numbers $\tilde{1}$ and $\tilde{0}$ as fuzzy positive ideal solution (PIS) and negative ideal solution (NIS). Chen's method sometimes violates the basic concepts of traditional TOPSIS. This study thus proposes fuzzy hierarchical TOPSIS, which not only is well suited for evaluating fuzziness and uncertainty problems, but also can provide more objective and accurate criterion weights, while simultaneously avoiding the problem of Chen's Fuzzy TOPSIS. For application and verification, this study presents a numerical example and build a practical supplier selection problem to verify our proposed method and compare it with other methods.

1. Introduction

Multi-criteria decision-making (MCDM) methods are formal approaches to structure information and decision evaluation in problems with multiple, conflicting goals. MCDM can help users understand the results of integrated assessments, including tradeoffs among policy objectives, and can use those results in a systematic, defensible way to develop policy recommendations. MCDM methods have been widely used in many research fields. Different approaches have been proposed by many researchers, including the Analytic Hierarchy Process (AHP) [20], Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [13] and MCDM [13,18]. According to Løken [16], existing MCDA methods can be classified into three broad categories:

(1) Value measurement models

AHP and multiattribute utility theory (MAUT) are the best known method in this group.

(2) Goal, aspiration and reference level models

Goal programming (GP) and TOPSIS are the most important methods that belong to the group.

(3) Outranking models

ELECTRE and PROMETHEE are two main families of method in this group.

AHP was first proposed by Saaty [20], and has been applied in several areas of social sciences and management, such as project management [15], and military applications [6], etc. AHP integrates experts' opinions and evaluation scores, and devises the complex decision-making system into a simple elementary hierarchy system. The evaluation method in terms of ratio scale is then employed to perform relative importance pair-wise comparison among every criterion. This method decomposes complicated problems from higher hierarchies to lower ones.

The concept of TOPSIS is that the most preferred alternative should not only have the shortest distance from the positive ideal solution (PIS), but should also be farthest from the negative ideal solution (NIS). Hwang and Yoon [13] also described the TOPSIS concept, referring to the positive and negative ideal solutions as the ideal and anti-ideal solutions, respectively. Numerous applications of TOPSIS exist, including airline performance evaluating [2] and optimal material selection [14].

AHP and TOPSIS possess advantages in that they are easy to compute and easily understood, because the methods are directly giving a definite value by experts to calculate their final results. Though AHP is designed to capture expert knowledge/opinions, the conventional AHP does not reflect human thinking style. The

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linguistic expressions of fuzzy theory are regarded as natural representations of preferences/judgments. Characteristics such as satisfaction, fairness and dissatisfaction indicate the applicability of fuzzy set theory in capturing the preference structures of decision makers, while fuzzy set theory aids in measuring the ambiguity of concepts associated with subjective human judgments. Fuzzy MCDM theory thus can strengthen the comprehensiveness and reasonableness of the decision-making process.

Decision makers usually are more confident making linguistic judgments than crisp value judgments. This phenomenon results from inability to explicitly state their preferences owing to the fuzzy nature of the comparison process. Many studies have continually introduced the fuzzy concept to improve MCDM and solve linguistic and cognitive fuzziness problems. For example, fuzzy theory and AHP are combined to become the Fuzzy AHP (FAHP) method [3,4,6,19], which is a fuzzy extension of AHP, and was developed to solve hierarchical fuzzy problems. FAHPs are systematic approaches to the alternative selection and justification problem that use the concepts of fuzzy set theory and hierarchical structure analysis. FAHP can be applied to measure fuzzy linguistic cognition, and suffers form the disadvantage of unstable (i.e., non-unique) results being obtained by different defuzzification methods, and the ordering of alternatives will arise ranking reversion.

Chen [7] extended TOPSIS to fuzzy environments; this extended version used fuzzy linguistic value (represented by fuzzy number [11]) as a substitute for the directly given crisp value in grade assessment. This modified TOPSIS is a practical method and fits human thinking under actual environment. The criteria weighting of Chen are directly provided by experts, and the expert weightings are then averaged, also the calculated results sometimes violate the basic concept of TOPSIS. Therefore, this study proposes fuzzy hierarchical TOPSIS, which uses simplified parameterized metric distance and FAHP to modify Chen's Fuzzy TOPSIS to overcome the above disadvantages.

The remainder of this paper is organized as follows: Section 2 briefly describes the metric distance method, and the parameters of metric distance, linguistic variable and defuzzification. Next, Section 3 simplifies the parameterized metric distance and proposes an algorithm of fuzzy hierarchical TOPSIS, and illustrates a numerical example to verify the proposed method. Section 4 then presents a practical problem involving supplier selection to verify the proposed method and compare it with other methods. The final section presents conclusions.

2. Preliminary

This section briefly describes the metric distance method, and the parameters of metric distance, linguistic variable and defuzzification.

2.1. Metric distance method

This study used the metric distance method of Chen and Cheng [5,8] to calculate the dispersion between each point and FPIS and FNIS. The method of Chen and Cheng thus is briefly reviewed below:

A metric space $\langle X, \rho\rangle$ in which X is a nonempty and ρ is a real number function, both X and ρ are on the $X \times X$, where x, y, and $z \in X$:

$$\begin{cases} \text{i.}\rho(x,y) \geq 0; \\ \text{ii.}\rho(x,y) = 0 \quad \text{if only if} \quad x = y; \\ \text{iii.}\rho(x,y) = \rho(y,x); \\ \text{iv.}\rho(x,y) \leq \rho(x,z) \end{cases} \tag{1}$$

Then function ρ is called a metric.

If two metric spaces $\langle X, \rho\rangle$ and $\langle Y, \omega\rangle$ form a new metric space called Cartesian product $X \times Y$, its point set is a set of $X \times Y = \{\langle x, y\rangle : x \in X, y \in Y\}$, and its metric τ is defined as follows:

$$\tau(<x_1, y_1>, <x_2, y_2>) = [\rho(x_1, x_2)^2 + \omega(y_1, y_2)^2]^{1/2} \tag{2}$$

For convenience, the fuzzy number can be denoted using $[a, b, c, d; 1]$, and the membership function (MF) $f_{\tilde{A}}$ of the fuzzy number $\tilde{A} = [a, b, c, d; 1]$, can be expressed as

$$f_{\tilde{A}} = \begin{cases} f_{\tilde{A}}^{\mathrm{L}}(x), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ f_{\tilde{A}}^{\mathrm{R}}(x), & c \leq x \leq d, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

where $f_{\tilde{A}}^{\mathrm{L}} : [a, b] \to [0, 1]$ and $f_{\tilde{A}}^{\mathrm{R}} : [c, d] \to [0, 1]$. Since $f_{\tilde{A}}^{\mathrm{L}} : [a, b] \to [0, 1]$ is continuous and strictly increasing, the inverse function of $f_{\tilde{A}}^{\mathrm{L}}$ exists. Similarly, $f_{\tilde{A}}^{\mathrm{R}} : [c, d] \to [0, 1]$ is continuous and strictly decreasing, the inverse function of $f_{\tilde{A}}^{\mathrm{R}}$ also exists.

From Cheng [5], the inverse functions of $f_{\tilde{A}}^{\mathrm{L}}$ and $f_{\tilde{A}}^{\mathrm{R}}$ are denoted by $g_{\tilde{A}}^{\mathrm{L}}$ and $g_{\tilde{A}}^{\mathrm{R}}$, respectively. Since $f_{\tilde{A}}^{\mathrm{L}} : [a, b] \to [0, 1]$ is continuous and strictly increasing, $g_{\tilde{A}}^{\mathrm{L}} : [a, b] \to [0, 1]$ is also continuous and strictly increasing. Similarly, if $f_{\tilde{A}}^{\mathrm{R}} : [c, d] \to [0, 1]$ is continuous and strictly decreasing, then $g_{\tilde{A}}^{\mathrm{R}} : [a, b] \to [0, 1]$ is also continuous and strictly decreasing; $g_{\tilde{A}}^{\mathrm{L}}$ and $g_{\tilde{A}}^{\mathrm{R}}$ are continuous on a closed interval [0, 1] and are integrable on [0, 1]. That is, both $\int_0^1 g_{\tilde{A}}^{\mathrm{L}}(y)\mathrm{d}y$ and $\int_0^1 g_{\tilde{A}}^{\mathrm{R}}(y)\mathrm{d}y$ exist.

From Eq. (3), the metric can be adapted to fuzzy numbers, and can be redefined as:

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}^{\mathrm{L}}(x) & \text{for} \quad x \leq m_1 \\ f_{\tilde{A}}^{\mathrm{R}}(x) & \text{for} \quad x \geq m_1 \end{cases}, \; f_{\tilde{B}}(x) = \begin{cases} f_{\tilde{B}}^{\mathrm{L}}(x) & \text{for} \quad x \leq m_2 \\ f_{\tilde{B}}^{\mathrm{R}}(x) & \text{for} \quad x \geq m_2 \end{cases}$$

then

$$D(\tilde{A}, \tilde{B}) = \left[\int_0^1 (g_{\tilde{A}}^{\mathrm{L}} - g_{\tilde{B}}^{\mathrm{L}})^2 \;\mathrm{d}y + \int_0^1 (g_{\tilde{A}}^{\mathrm{R}} - g_{\tilde{B}}^{\mathrm{R}})^2 \;\mathrm{d}y\right]^{1/2} \tag{4}$$

denotes the distance of $\tilde{A}(x)$ and $\tilde{B}(x)$, and the inverse functions of $f_{\tilde{A}}^{\mathrm{L}}$ and $f_{\tilde{A}}^{\mathrm{R}}$ are $g_{\tilde{A}}^{\mathrm{L}}$ and $g_{\tilde{A}}^{\mathrm{R}}$.

Eq. (4) is the generalized ranking fuzzy number method, and our method can be applied to the negative and positive ranking of fuzzy numbers. For convenience of ranking n positive fuzzy numbers $\tilde{A}_1, \tilde{A}_2, \cdots$, and $\tilde{A}_n$, let $\tilde{B}(x) = 0$ in Eq. (4), so that D can be rewritten as:

$$D(\tilde{A}_i, 0) = \left[\int_0^1 (g_{\tilde{A}_i}^{\mathrm{L}})^2 \;\mathrm{d}y + \int_0^1 (g_{\tilde{A}_i}^{\mathrm{R}})^2 \;\mathrm{d}y\right]^{1/2} \tag{5}$$

The ranking of the fuzzy number increases with the value of $D(\tilde{A}_i, 0)$.

2.2. The parameterized of metric distance

From the metric distance of Chen and Cheng [8], this study briefly describes the concept of minimum metric D and Eq. (10) to calculate the parameter of fuzzy mean $x_0 (m = x_0)$ and fuzzy standard deviation σ $(\alpha = \beta = \sigma)$ as follows:

Let $\tilde{A}$ denote is a generalized LR fuzzy number and let the inverse functions of $f_{\tilde{A}}^{\mathrm{L}}$ and $f_{\tilde{A}}^{\mathrm{R}}$ be $g_{\tilde{A}}^{\mathrm{L}} = (x_0 - \sigma) + \sigma y$ and $g_{\tilde{A}}^{\mathrm{R}} = (x_0 + \sigma) - \sigma y$. The symmetry function $\tilde{A}'$s of $S[x_0,\sigma]$ can be obtained by immunizing this metric D, namely:

$$D(\tilde{A}, S[x_0, \sigma]) = \int_0^1 (g_{\tilde{A}}^{\mathrm{L}} - S[x_0, \sigma]^L)^2 \;\mathrm{d}y + \int_0^1 (g_{\tilde{A}}^{\mathrm{R}} - S[x_0, \sigma]^{\mathrm{R}})^2 \;\mathrm{d}y \tag{6}$$