Int. J. Production Economics 115 (2008) 143–150

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Economic production quantity model for items with imperfect quality subject to learning effects

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ARTICLE INFO

Article history:
Received 9 May 2007
Accepted 25 May 2008
Available online 29 May 2008

Keywords:
EOQ
Imperfect quality
Learning curve

ABSTRACT

Salameh and Jaber [2000. International Journal of Production Economics 64, 59–64] developed an inventory situation where items received are not of perfect quality (defective), and after 100% screening, imperfect quality items are withdrawn from inventory and sold at a discounted price. This paper extends the work of Salameh and Jaber by assuming the percentage defective per lot reduces according to a learning curve, which was empirically validated by data from the automotive industry. Mathematical models are developed with numerical examples provided and discussed. The developed model was compared with the model of Salameh and Jaber to emphasize the importance of learning.

1. Introduction

The economic order quantity (EOQ) is the first and the simplest model available in the inventory literature. It has been the cornerstone for numerous models with a reasonably good survey provided in Silver et al. (1998). The popularity of the EOQ model is perhaps attributed to its simple mathematics, which allows managers to compute their order quantities using a business calculator. The simplicity of the mathematics is the result of the assumptions in developing the EOQ model, which are viewed by many as being unrealistic (e.g., Jaber et al., 2004). One of these assumptions is that shipments of raw materials or components received by a buyer conform to quality specification and therefore contains no defects.

Salameh and Jaber (2000) extended the EOQ model assuming a random yield where each lot size shipment contains a random fraction of imperfect quality items with a known probability distribution. They assumed that each shipment undergoes a 100% screening and items nonconforming to quality standards are sold at a discounted price as a single batch at the end of the screening process, and therefore are instantly removed from inventory. Several studies extended the work of Salameh and Jaber (2000) reflecting the attention it has received. Brief surveys of these studies are provided below.

Recently, Papachristos and Konstantaras (2006) examined the model of Salameh and Jaber (2000) closely and discussed many of its assumptions, and in particular, those aimed at avoiding shortages. Huang (2004) and Chung and Huang (2006) investigated the model of Salameh and Jaber (2000) in a two-level supply chain (vendor–buyer), while Wee et al. (2007) and Eroglu and Ozdemir (2007) independently extended it by allowing for shortages. In addition, Chan et al. (2003) develop an economic production model using similar assumptions as Salameh and Jaber (2000), and, likewise, Chang (2004) develop an EOQ model with fuzzy defective rate and demand. The authors' survey of the inventory literature reveals that there is no published work that investigates the model of Salameh and Jaber (2000) for learning effects.

Empirical data collected from an automotive manufacturer for shipments of raw material revealed that the

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0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2008.05.007

percentage of defective items per lot decreases with cumulative number of shipments conforming to a learning curve. This is not surprising knowing that some studies reported that quality improves because of learning (e.g., Dolinsky et al., 1990; Lapré et al., 2000; Jaber and Bonney, 2003). Therefore, this paper extends the work of Salameh and Jaber (2000) by assuming the percentage of defective items per shipment reduces according to a learning curve.

The remainder of this paper is organized as follows. Section 2 presents a brief introduction to the learning curve theory and discusses its form. Section 3 is for assumptions and notations. Section 4 is mathematical modeling. Section 5 provides numerical examples and discussion of results. The paper summarizes and concludes in Section 6.

2. The learning curve

The earliest learning curve representation is a geometric progression that expresses the decreasing cost required to accomplish any repetitive operation. The theory in its most popular form states that as the total quantity of units produced doubles, the cost per unit declines by some constant percentage (e.g., Yelle, 1979; Jaber, 2006).

The form of the learning curve (e.g., power versus exponential) has been debated by several authors; refer to Jaber (2006) for discussion. There is almost unanimous agreement among practitioners and academicians that the learning curve is best described by a power as suggested by Wright (1936). It is worth noting that the learning curve in practice is an 'S'-shaped curve (Jordan, 1958; Carlson, 1973), as described in Fig. 1. The first phase (incipient) is the phase during which the worker is getting acquainted with the set-up, the tooling, instructions, blueprints, the workplace arrangement, and the conditions of the process. In this phase improvement is slow. The second phase (learning) is where most of the improvement, e.g., reduction in errors, changes in workplace, changes in the distance moved takes place. The third and last phase (maturity) represents the levelling of the curve.

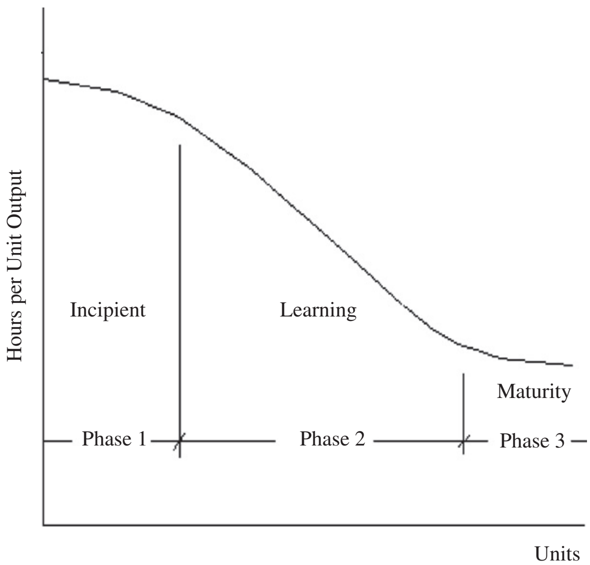

Fig. 1. The three phases of the learning curve (Jordan, 1958).

Several learning curve models were fitted to the collected data and the S-shaped logistic learning curve was found to fit well, and it is of the form

$$p(n) = \frac{a}{g + e^{bn}} \tag{1}$$

where a, b, and $g>0$ are the model parameters, n is the cumulative number of shipments, and $p(n)$ is the percentage defective per shipment n. The fit parameters are $a = 70.067$, $b = 0.7932$, $g = 819.76$, where the sum square of errors (SSE) is 0.001442, with the fit of $p(n) = 70.067/(819.76 + e^{0.7932n})$ to empirical data is shown in Fig. 2.

The data were also fitted to the Wright's learning curve, $p(n) = p_0 n^{-\beta}$, and the fit parameters are $p_0 = 0.3461$, $\beta = -1.4513$, where the SSE is 0.07867. However, by separating the data into two sets (observations 1–5 and 6–16), the power form (Wright's) learning curve fit the second dataset (learning and maturity) well confirming the behavior described in Fig. 1 (Jordan, 1958). The fit parameters are $p_0 = 0.1679$, $\beta = -1.6717$, where the SSE is 0.008603. This suggests that the Wright's learning curve is appropriate for the learning phase and not the incipient phase. However, there are industrial situations where the incipient phase is significantly short making the Wright learning curve an appropriate model to use (e.g., Dar-El, 2000).

3. Assumption and notations

3.1. Assumptions

The following assumptions are being made for developing the mathematical models: (1) demand rate is constant, (2) shortages are not allowed, (3) lead time is zero, (4) 100% screening of items is done each shipment, (5) defective items are sold at a discounted price, (6) percentage defectives follows a learning curve, and (7) time horizon is infinite/finite.

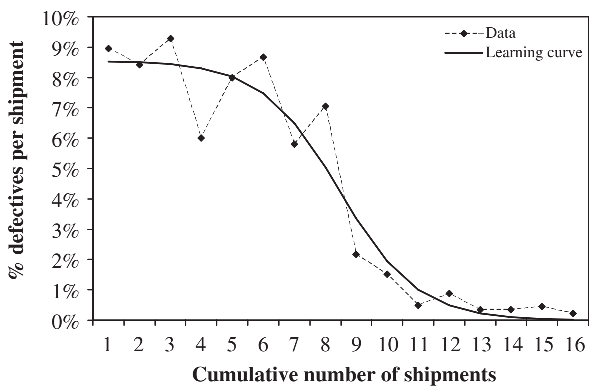

Fig. 2. The percentage defective per shipment follows a learning curve.