Expert Systems with Applications 36 (2009) 8892–8899

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Supplier selection in a fuzzy group setting: A method using grey related analysis and Dempster–Shafer theory

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A R T I C L E I N F O

Keywords:
Fuzzy group decision making
Grey related analysis
Dempster–Shafer rule of combination
International supplier selection

A B S T R A C T

Supplier selection in a fuzzy group setting is a very important strategic decision involving decisions balancing a number of conflicting criteria and opinions from different experts. This paper uses grey related analysis and Dempster–Shafer theory to deal with this fuzzy group decision making problem. First, in the individual aggregation, grey related analysis is employed as a means to reflect uncertainty in multi-attribute models through interval numbers. Second, in the group aggregation, the Dempster–Shafer (D–S) rule of combination is used to aggregate individual preferences into a collective preference, by which the candidate alternatives are ranked and the best alternative(s) are obtained. The proposed approach uses both quantitative and qualitative data for international supplier selection. It provides alternative tools to evaluate and improve supplier selection decisions in an uncertain global market.

1. Introduction

Supplier selection involves the need to trade-off multiple criteria, as well as the presence of both quantitative and qualitative data. With the increase in outsourcing, offshore sourcing, and various electronic businesses, these decisions are becoming ever more complex. When more experts (and more conflicting opinions) are involved, this decision becomes a problem of selecting ideal suppliers in a fuzzy group setting.

Various studies have presented methods to aggregate different expert opinions in a fuzzy group decision making problem (Kulak & Kahraman, 2005; Zhang, Wu, & Olson, 2005). One of the key issues is how to choose the aggregation operators and functions to combine different degrees of membership into one numerical value. Expert opinions, a common way to hedge risk, can be represented by fuzzy numbers when subjectivity, vagueness and imprecision enter into those assessments (Bardossy, Duckstein, & Bogardi, 1993). A number of studies have used fuzzy preference relations (Fedrizzi & Kacprzyk, 1980; Kacprzyk & Fedrizzi, 1988; Kacprzyk, Fedrizzi, & Nurmi, 1992; Nurmi, 1981; Tanino, 1984, 1990), and Ishikawa et al. (1993) used an interval value to represent judgments of experts and derived a group consensus judgment from cumulative frequency distribution. Recently, Hsu and Chen (1996) proposed a method called *similarity aggregation* to aggregate individual opinions of experts. To date, more than ninety families of operators have been extensively studied (Beliakov & Warren, 2001).

Of these, TOPSIS (order preference by similarity to ideal solution) (Hwang & Yoon, 1981) and LINMAP (linear programming for multidimensional analysis of preference) (Srinivasan & Shocker, 1973) are the two functions most relevant to this paper. TOPSIS computes both a positive ideal solution (PIS) and a negative ideal solution (NIS) from the decision matrix and then generates the best compromise alternative with the shortest distance to PIS and the farthest from NIS. The LINMAP method uses pair-wise comparisons of alternatives given by decision makers and generates the best compromise alternative as the solution that has the shortest distance to the positive ideal solution. Both TOPSIS and LINMAP try to generate the best compromise alternative as the solution nearest to the PIS, as does Deng's grey related analysis (Deng, 1982). Grey system theory addresses the concept that information is sometimes incomplete or unknown. The intent is the same as in factor analysis, cluster analysis, and discriminant analysis, except that those methods often do not work well when sample size is small and sample distribution is unknown. Interval numbers are standardized through norms, which allow transformation of index values through product operations (Zhang et al., 2005). Grey system theory provides another tool to reflect uncertainty into the multi-attribute model, using reference number sequence rather than the ideal solution of TOPSIS and LINMAP.

This paper proposes a hybrid method using Deng's grey system theory and Dempster–Shafer evidence theory (Dempster, 1966, 1967, 1968; Shafer, 1976). The proposed grey-evidence hybrid solves fuzzy group decision-making problems in two phases: First, grey related analysis is used to reflect uncertainty between experts in multi-attribute models through interval numbers, generating individual preferences. Second, the Dempster–Shafer (D–S) rule

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0957-4174/$ - see front matter © 2008 Published by Elsevier Ltd.
doi:10.1016/j.eswa.2008.11.010

of combination is used to aggregate individual preferences into collective preferences by which the best alternative(s) are obtained.

Compared to probability theory, D–S evidence theory captures more information to support decision-making, by identifying the uncertain and unknown evidence. It provides a mechanism to derive solutions from various vague evidences without knowing much prior information. Combining both theories enables the ultimate decision makers to take advantage of both methods merits and use experts to deal with uncertainty and risk confidently.

Using the proposed approach, we solved a complex international supplier selection problem in the fuzzy group setting, taking both quantitative and qualitative data into consideration. The results show that the proposed approach simplifies and speeds up trade-off analysis among costs, quality acceptance levels, on-time delivery, and risk factors, and opens up the possibility of a family of tools to evaluate and improve other selection decisions in an uncertain global market.

The rest of the paper is organized as follows: we start with reviewing the Dempster–Shafer theory of combination in Section 2. Section 3 delivers the method and Section 4 demonstrates the application of the proposed method to international supplier selection problem. Section 5 concludes the paper.

2. Dempster–Shafer theory of combination

Evidence theory (Fedrizzi & Kacprzyk, 1980; Kacprzyk & Fedrizzi, 1988; Kacprzyk et al., 1992; Thierry, 1997) is a branch of mathematics of uncertain reasoning and it is developed to represent outcomes of statistical experiments by the interval-valued probabilities (Walley, 1991). It aims to combine unexpected empirical evidence in an individual's mind and thus constructs a coherent picture of reality. This section presents the fundamentals of D–S theory of evidence. The readers may refer to (Fioretti, 2004; Thierry, 1997) for details. Let Θ be a finite set of mutually exclusive and exhaustive hypotheses, called the *frame of discernment*. For example, in a multi-attribute group decision-making problem, Θ might be a set of alternatives under evaluation. A function m is called a *basic belief assignment* (BBA) if and only if it is satisfied by

$$m : 2^{\Theta} \to [0,1]$$

$$m(\emptyset) = 0$$

$$\sum_{A \subseteq \Theta} m(A) = 1$$

For any $A \subseteq \Theta$, $m(A)$ represents the belief that one is willing to commit exactly to A, given a certain piece of evidence. The subset A of Θ is called the *focal elements* of m if $m(A) > 0$. The plausibility (Pls) and belief (Bel) functions are two main definitions used to represent the imprecision and uncertainty in the decision-making process. The Pls and Bel of hypothesis A $(A \subseteq \Theta)$, also known as lower and upper probability functions, are computed by the mass function:

$$\mathrm{Bel}(A) = \sum_{A \subseteq B} m(A) \tag{1}$$

$$\mathrm{Pls}(A) = \sum_{A \cap B \neq \emptyset} m(A) \tag{2}$$

The quantity $\mathrm{Bel}(A)$ can be interpreted as a global measure of one's belief that the hypothesis is true, while $\mathrm{Pls}(A)$ is the amount of belief that could *potentially* be placed in A, if further information becomes available. Thus, the magnitude of imprecision of hypothesis A is captured by the belief interval $[\mathrm{Bel}(A), \mathrm{Pls}(A)]$.

In reality, a decision maker can often gain access to more than one information source in order to make his/her decisions. The evidence theory constructs a set of hypotheses of known mass values from these information sources and then computes a new set of numbers m that represents combined evidence. This construction rule is called *Dempster–Shafer rule of combination* for group aggregation.

Suppose there are two BBAs m_1 and m_2 on Θ, induced by two independent items of evidence. If $\sum_{A_i \cap B_j = \emptyset} m_1(A_i) m_2(B_j) < 1$, which indicates they are *combinable* and the orthogonal sum is commutative and associative, then m_1 and m_2 can be combined by use of the *D–S rule of combination* to a new BBA, i.e., the orthogonal sum:

$$\begin{cases} m(C) = m_1 \oplus m_2(C) = \dfrac{\sum_{A_i \cap B_j = C} m_1(A_i) m_2(B_j)}{1 - \sum_{A_i \cap B_j = \emptyset} m_1(A_i) m_2(B_j)} & C \neq \emptyset \\ m(C) = 0 & C = \emptyset \end{cases} \tag{3}$$

If $\sum_{A_i \cap B_j = \emptyset} m_1(A_i) m_2(B_j) = 1$, then two bodies of evidence are completely contradictory and the Dempster–Shafer rule (3) does not apply: in this situation, they can be treated as one single body of evidence over alternative possibilities.

Let us use a simple example to explain the D–S rule of combination: denote by m_1 and m_2, two pieces of evidence over a frame of discernment $\{A,B\}$, which consists of two singular elements, A and B. The BBAs are $m_1(A) = 0.75$, $m_1(B) = 0.5$, $m_2(A) = 0.5$, and $m_2(B) = 0.5$. Combining the two evidence using D–S rule of combination leads to:

$$m(A) = \frac{m_1(A) * m_2(A)}{1 - k} = 1 \quad \text{and} \quad m(B) = \frac{m_1(B) * m_2(B)}{1 - k} = 0.67,$$

where $k = m_1(A) * m_2(B) + m_1(B) * m_2(A) = 0.625$.

Having summarized all the available evidence in the form of a BBA, we then turn to the problem of selecting the dominant alternatives. To do this, one simple approach is to adopt the *pignistic* probability distribution (Tanino, 1984) BetP defined for all $C \subseteq \Theta$ as

$$\mathrm{BetP}(C) = \sum_{C \subseteq \Theta} \frac{m(C)}{|C|} \tag{4}$$

where $|C|$ denotes the cardinality of C $(C \subseteq \Theta)$. The *pignistic* function in (4) is used to designate a probability function constructed from a belief function.

In a multi-attribute group decision-making problem with M alternatives, (4) can be rewritten as follows:

$$\mathrm{BetP}(C) = m(C) + \frac{m(\Theta)}{M} \tag{5}$$

where C denotes a certain alternative to be evaluated and $m(\Theta)$ refers to the BBA of the whole *frame of discernment*, i.e., BBA of unknown information. An alternative with a larger *pignistic* function value is preferred to one with smaller value.

3. The approach

This section presents the proposed approach using grey related analysis and Dempster–Shafer rule of combination. I discuss a technique to transform fuzzy numbers into interval numbers and then present the body of the approach.

Note that fuzzy numbers can easily be transformed into interval numbers using α-cut technique. Here we base our discussion on interval numbers for simplicity. For example, a trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ depicted in Fig. 1 can be transformed into an interval number $a = [\alpha * a_2 + (1-\alpha) * a_1, \alpha * a_3 + (1-\alpha) * a_4]$ by use of the α-cut technique (Dubois & Prade, 1980; El-Hawary, 1998).

Consider a group decision-making problem with fuzzy number. Let $X = (X_1, X_2, \ldots, X_m)$ be a finite set of alternatives (candidates) and $D = (D_1, D_2, \ldots, D_s)$ be a set of decision makers. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_s)^T$ be the weight vector of decision makers,