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The economic production quantity (EPQ) with shortage derived algebraically

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Abstract

Previously, in several papers and textbooks, the classical economic order quantity (EOQ) and the economic production quantity (EPQ) formulas for the shortage case, have been derived using differential calculus and solving two simultaneous equations (derived from setting the two first partial derivatives to zero) with the need to prove optimality conditions with second-order derivatives. In a previous original piece of work, a new approach to find the EOQ with backlogging using some slight algebraic developments appeared. This paper extends the mentioned algebraic approach to the EPQ formula taking shortages into consideration within the case of only one backlog cost per unit and time unit. The final expressions provide the same formulas that are available in the classic textbooks on inventory theory.

Keywords: Economic order/production quantity; Shortage

1. Introduction

Economic lot size models have been studied extensively since Harris [1] first presented the famous economic order quantity (EOQ) formula. Much of the literature on inventory theory contains the basic models of EOQ with/without shortages, and economic production quantity (EPQ) with/without shortages. To obtain the final expressions of these models for determining the economic order/manufacturing quantity require a mathematical methodology which is experienced as complicated for many undergraduate students. However, some authors have presented several approaches to find the EOQ without shortage in a simpler manner. For example, Thierauf and Grosse [2, pp. 187–193] presented a tabular, a graphical and an algebraic approach as alternatives to the differentiation approach.

In a recent paper, Grubbström and Erdem [3] showed that the formulae for the standard EOQ with backlogging could be derived without the classical technique of optimisation, in other words, without differential calculus.

Grubbström and Erdem mentioned that this approach must be considered as a pedagogical advantage for explaining the EOQ concepts to students that lack knowledge of derivatives, simultaneous equations and the procedure to construct

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and examine the Hessian matrix. This approach could therefore be used to introduce the inventory theory at the high school level.

This paper extends Grubbström and Erdem's method to algebraically determine the EPQ with shortage in the case of only one shortage cost per unit and time unit.

2. Algebraic derivation of the EPQ model formulae

This section includes the algebraic derivation of the EPQ model with shortage under the assumption of a finite production rate. The following notation is used in the derivation of the total average cost function.

$I_{\max}$ = maximum on-hand inventory level (units),
B = maximum shortage (backorder) level (units),
h = inventory carrying cost per unit and time unit ($h = ic$),
b = shortage cost per unit short and time unit,
D = demand rate, units per time,
P = production rate, units per time ($P > D$),
K = setup cost of production (fixed cost),
c = unit variable cost of production,
i = inventory carrying rate,
Q = economic production quantity (EPQ),
C = total average cost of the inventory system.

Fig. 1 shows the behavior of the EPQ inventory system that will be analysed algebraically.

The total average cost per time unit is the sum of the setup cost, carrying cost, shortage cost and item cost, and it has the following form:

$$C = \frac{KD}{Q} + h\left\{\frac{[Q(1-D/P)-B]^2}{2Q(1-D/P)}\right\} + \frac{bB^2}{2Q(1-D/P)} + cD. \tag{1}$$

Let $\rho = (1 - D/P)$ to obtain

$$C = \frac{KD}{Q} + h\left[\frac{(Q\rho - B)^2}{2Q\rho}\right] + \frac{bB^2}{2Q\rho} + cD. \tag{2}$$

The maximum inventory level on-hand is

$$I_{\max} = Q\rho - B. \tag{3}$$

Then,

$$Q\rho = I_{\max} + B, \tag{4}$$

or,

$$Q = \frac{I_{\max} + B}{\rho}. \tag{5}$$

Substituting Eqs. (3)–(5) into Eq. (2) yields

$$C = \frac{KD\rho}{I_{\max} + B} + \frac{h(I_{\max})^2}{2(I_{\max} + B)} + \frac{bB^2}{2(I_{\max} + B)} + cD$$
$$= \left[\frac{D}{I_{\max} + B}\right]\left[\frac{h(I_{\max})^2}{2D} + \frac{bB^2}{2D} + K\rho\right] + cD. \tag{6}$$

Now, $K\rho$ can be rewritten as the following expression (similar to Grubbström and Erdem's

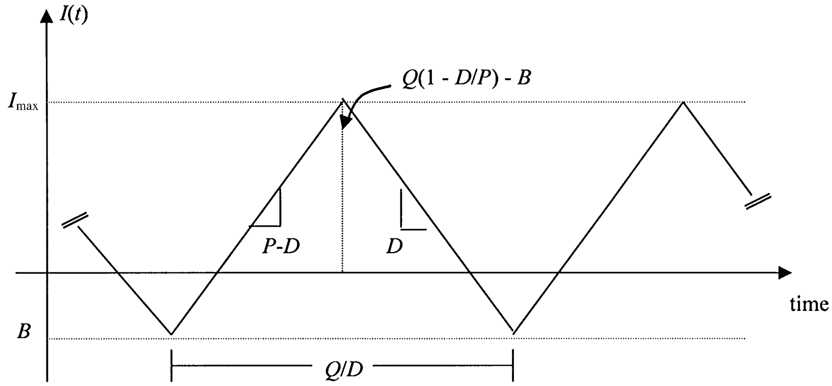

Fig. 1. Single-product model with constant demand rate and constant production rate.