Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Applied Mathematical Modelling 29 (2005) 987–1003
APPLIED MATHEMATICAL MODELLING
www.elsevier.com/locate/apm

# Deterministic economic production quantity models with time-varying demand and cost

Jinn-Tsair Teng [a,*], Liang-Yuh Ouyang [b], Chun-Tao Chang [c]

[a] Department of Marketing and Management Sciences, Cot Sakos College of Business, The William Paterson University of New Jersey, Wayne, NJ 07470, USA
[b] Graduate Institute of Management Sciences, Tamkang University, Tamsui, Taipei 25137, Taiwan, R.O.C.
[c] Department of Statistics, Tamkang University, Tamsui, Taipei 25137, Taiwan, R.O.C.

Received 1 March 2003; received in revised form 1 December 2004; accepted 8 February 2005
Available online 19 March 2005

## Abstract

In today's time-based competition, the unit cost of a high-tech product declines significantly over its short product life cycle. Consequently, in this paper, we relax the traditional economic production quantity model to allow for time-varying cost. We then prove that the optimal production schedule uniquely exists. In addition, we also show that the total cost is a convex function of the number of replenishments, which reduces the search for the optimal solution to finding a local minimum. Furthermore, we characterize the influences of both demand and cost over the length of production run time and the economic production quantity.


*Keywords:* Production/Inventory; Lot-size; Time-varying cost; Time-varying demand

* Corresponding author. Tel.: +1 973 720 2651; fax: +1 973 720 2809.
*E-mail address:* tengj@wpunj.edu (J.-T. Teng).

0307-904X/$ - see front matter 
doi:10.1016/j.apm.2005.02.002

## 1. Introduction

The classical economic production quantity (EPQ) model is widely used principally because it is so simple to use and apply. However, a major problem in using the EPQ is that it assumes not only a constant demand rate but also a fixed unit purchasing cost. As we know from a product life cycle, the demand rate remains stable only in the maturity stage. Moreover, in time-based competition today, the unit cost of a high-tech product declines significantly over its short product life cycle. For example, the cost of a personal computer drops constantly as shown in Lee et al. [1]. Therefore, using the EPQ formulation in stages other than the maturity stage or for a product with short product life cycle will cause varying magnitudes of error. In addition, the cost of purchases as a percentage of sales is often substantial (52% for all industry) as shown in Heizer and Render [2]. Consequently, adding the purchasing strategy into EPQ model is vital. For example, when the gasoline price is costly going up, we adjust it by driving less (or buying fuel-saving cars), and vice versa. Similarly, a manufacturer needs to adjust its production strategy when the cost is fluctuating with time.

In reality, the demand may vary with time. One method of dealing with EPQ models with time-varying demand over a finite-planning horizon is the use of discrete dynamic programming (e.g., [3]). Based on our decades of teaching experiences, students do not have any difficulty learning the continuous version of EPQ (or linear programming). However, there are many students who have difficulty handling tedious and cumbersome dynamic programming (or integer programming). As stated in Friedman [4], "In particular, Wagner and Whitin use this approach (i.e., dynamic programming) to formulate a dynamic version of the economic lot size model. Although this may be a satisfactory approach, it is generally preferable to solve analytically for the optimal replenishment policy, whenever possible". As a result, for easy understanding and applying, we will solve the EPQ problem here by a continuous version with a simple analytical solution, instead of using a discrete version of dynamic programming. In the growth stage of a product life cycle, the demand rate can be well approximated by a linear form. Resh et al. [5] proposed an algorithm to find the optimal replenishment number and time scheduling for time-proportional demand (i.e., $f(t) = bt$, with $b > 0$). Concurrently, Donaldson [6] also derived an analytical solution to a similar model in which the demand trend is linear (i.e., $f(t) = a + bt$, with $a, b \neq 0$). Barbosa and Friedman [7] further generalized the solutions for various power-form demand rates (i.e., $f(t) = bt^r$, with $b > 0$, $r > -2$). Henery [8] then extended the demand function to be any log-concave form (i.e., $f(t)$ is log-concave). Recently, Teng et al. [9] further generalized the inventory lot-size models to allow for fluctuating demand (which is more general than constant, increasing, decreasing, and log-concave demand patterns). By contrast to above EOQ models, Balkhi [10] first generalized EPQ model for deteriorating items in which the demand, production and deterioration rates are continuous functions of time. Lately, Goyal and Giri [11] extended Balkhi's EPQ model to allow for partial backlogging, and compared two different possible models. Other related papers were written by Benkherouf and Balkhi [12], Dave [13], Goyal et al. [14], Hariga [15], Hariga and Goyal [16], Silver et al. [17], Teng et al. [18,19], and Yang et al. [20].

All of the above models assume that the purchase (or production) cost is constant. In contrast, we first assume, for generality, that the unit production cost is positive and fluctuating with time. In addition, we also assume that the demand function is positive and fluctuating with time. As a result, our proposed model is suitable for today's high-tech products during any given time