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An elaborative unit cost structure-based fuzzy economic production quantity model

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Abstract

The purpose of this paper is to investigate and propose a fuzzy extended economic production quantity model based on an elaboratively modeled unit cost structure. This unit cost structure consists of the various lot-size correlative components such as on-line setups, off-line setups, initial production defectives, direct material, labor, and depreciation in addition to lot-size non-correlative items. Thus, the unit cost is correlatively modeled to the production quantity. Therefore, the modeling or the annual total cost function developed consists of not only annual inventory and setup costs but also production cost. Moreover, via the concept of fuzzy blurred optimal argument and the vertex method of the α-cut fuzzy arithmetic (or fuzzy interval analysis), two solution approaches are proposed: (1) a fuzzy EPQ and (2) a compromised crisp EPQ in the fuzzy sense. An optimization procedure, which can simultaneously determine the α-cut-vertex combination of fuzzy parameters and the optimizing decision variable value, is also proposed. The sensitivity model for the fuzzy total cost and thus EPQ to the various cost factors is provided. Finally, a numerical example with the original data collected from a firm demonstrates the usefulness of the new model.

Keywords: Economic production quantity; Inventory model; Unit cost structure; Fuzzy interval analysis; Fuzzy arithmetic; Fuzzy function optimization

1. Introduction

The economic production/order quantity (EPQ/EOQ) model was first introduced in the earlier decades of the last century. Since then, it is widely accepted by many industries [1] and today many different variations have been solved (e.g., see [2,3]). More recently, to cope with the uncertainty or fluctuation problems in the human subjectively originated data, fuzzy EPQ/EOQ models have been proposed using fuzzy set theory. In the treatment of fuzzy EPQ/EOQ models, two directions exist:

(1) The total cost function is viewed as a crisp mapping from some fuzzy variable (production quantity) and thus yields fuzzy expressions [4,5]. In other words, the fuzziness originates only with the lot size;
(2) The mapping itself is fuzzy with fuzzy parameters (e.g. [6–8]) and thus blurs the image of some crisp argument (e.g., production quantity).

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In the second direction, most models have focused only on the directly defuzzified version of the fuzzy total cost function to obtain a compromised crisp EPQ/EOQ. This approach ignores the fact that in a fuzzy environment, the decision of EPQ/EOQ is also blurred. Vujosevic et al. [7] considered four approaches to solving the fuzzy EOQ, two of which involved true minimization of fuzzy total cost. Yet, one of them apparently is a directly fuzzified version of the conventional crisp EOQ formula rather than being derived from the fuzzy total cost function. Therefore, there exists a fuzzy mathematical error (Appendix A). In this paper, by using the concept of fuzzy blurred optimal argument on a nonfuzzy domain and the vertex method of α-cut fuzzy arithmetic (or fuzzy interval analysis), a newly devised and extended EPQ model is proposed. Furthermore, a new algorithm and a corrective algorithm for the case where local extrema in the fuzzy EPQ exists that enable us to simultaneously obtain the combination of α-cut vertices of the fuzzy parameters and production quantity are also proposed.

In this newly extended EPQ model, the unit cost structure and its relationships to the inventory cost and total producing cost is thoroughly examined. In establishing the EPQ model, the inventory holding cost in which only a representative overall unit cost and setup cost are used often does not represent fully the essence of the inventory problem. The various cost components that contribute to a unit cost, such as direct materials and labor, energy, defectives, and part of equipment depreciation, etc., are all correlated to the production quantity. Consequently and inclusively the unit cost, unit-inventory carrying cost, and annual producing cost are all related to the production quantity. Thus, a more elaboratively modeled unit cost structure, which results in an EPQ model that extends and differs from the traditional EPQ model in including the elaboratively modeled unit cost and annual production cost, is needed and is proposed here.

Moreover, these costs may occur in a multi-stage production case where the stages may have different levels of influence. The importance of multi-process consideration in an EPQ problem was discussed in [9] in a WIP inventory, material and product lot-size problem. Such consideration can lead to an insight into the true needs of process improvements and/or goals, which is also considered in this paper. With the above considerations, the proposed model is also extended to the sensitivity analysis mode by incorporating extra parameters that indicate a firm's process improvement goal and/or managerial capabilities in controlling these cost factors. The resulting model is termed the "fuzzy reinforced EPQ model".

The remainder of this paper is organized as follows: Section 2 presents the model development or total cost function, before the fuzzy set theory is applied. Section 3 then fuzzifies the total cost function with the proposed solution approaches. Section 4 presents the fuzzy reinforced version. Section 5 gives a numerical example illustrating the new model with the original data collected from a firm located in Taiwan. Finally, some conclusions and suggestions for possible future research directions are discussed.

2. Model development—total cost function

Notations

q	production quantity per lot; with original lot size denoted as q_0
c	unit cost (cost per unit product produced) with q; with unit cost under q_0 denoted as c_0
d	annual demand in units
n_0	annual number of production cycles under q_0, i.e., d/q_0
r	demand quantity per day
p	production quantity per day
t_i	production cycle time of process i under q_0 not including on-line setup
s_i	percentage of unit cost from initial production scraps of process i under q_0
b_i	percentage of unit cost from direct labor cost of process i under q_0
e_i	percentage of unit cost from facility/equipment depreciation and energy consumption of process i under q_0
o_i	time of on-line setup of process i per cycle
f_i	time of off-line setup of process i per cycle
ft_i	ratio of f_i to t_i of process i under q_0
h	unit inventory cost as a percentage of the unit inventory value