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Economic production quantity with rework process at a single-stage manufacturing system with planned backorders

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ABSTRACT

In traditional inventory models such as the economic order quantity (EOQ) and the economic production quantity (EPQ) the sole objective is to minimize the total inventory-related costs, typically holding cost and ordering cost. These models do not consider the presence of defective products in the lot or rework of them. Recently, Jamal, Sarker, and Mondal (Jamal, A. A. M., Sarker, B. R., & Mondal, S., (2004). Optimal manufacturing batch size with rework process at single-stage production system. *Computers and Industrial Engineering*, *47*(1), 77–89) proposed a model, which dealt with the optimum batch quantity in a single-stage system in which rework is done by addressing two different operational policies to minimize the total system cost, but their models do not consider planned backorders. In this direction, this paper develops an EPQ type inventory model with planned backorders for determining the economic production quantity for a single product, which is manufactured in a single-stage manufacturing system that generates imperfect quality products, and all these defective products are reworked in the same cycle. We also establish the range of real values of the proportion of defective products for which there is an optimal solution, and the close form for the total cost of inventory system. The use of the inventory model is illustrated with numerical examples. The classical EOQ, EPQ inventory models with or without planned backorders and Jamal, Sarker and Mondal's model (Jamal, A. A. M., Sarker, B. R., & Mondal, S., (2004). Optimal manufacturing batch size with rework process at single-stage production system. *Computers and Industrial Engineering*, *47*(1), 77–89) are shown to be special cases of the EPQ inventory model presented in this paper.

1. Introduction

Economic lot size models have been studied extensively since Harris (1913) presented the famous EOQ formulae in 1913. Then in 1918, the economic production quantity (EPQ) inventory model was proposed by Taft (1918). In traditional inventory models such as the economic order quantity (EOQ) and the economic production quantity (EPQ) the sole objective is to minimize the total inventory-related costs, typically holding cost and ordering cost. Those models do not consider the presence of defective products in the lot and rework of them. The EOQ and EPQ inventory models have been used in many practical applications because these inventory models are simple and easy to implement in organizations. In addition, these models have one important feature which is their robustness with respect to small changes in the parameters. But the EOQ and EPQ inventory models have several assumptions that are very restrictive; one of them is that the manufacturing process necessarily generates good quality products. In fact, there are many situations in practice where the manufacturing process produces defective products due to reasons such as imperfect raw materials, skill level of the workers, machine capability, maintenance policies, or when the manufacturing process goes out of control, among others. However, many researchers and practitioners have developed inventory models for modeling different real world situations, two of which are the cases of economic order quantity (EOQ) and economic production quantity (EPQ) with planned backorders.

Consider the first unrealistic assumption of the classical EOQ/EPQ inventory models, that all units manufactured or purchased are good quality: we know that it is difficult to manufacture products with perfect quality because manufacturing processes are faulty (see for example Zipkin, 2000, p. 58; or Hayek and Salameh (2001), p. 585). There are several works related to the EOQ/EPQ inventory models that consider defective products. For example, Shih (1980) developed two inventory models where the proportion of defective products in the accepted lot is a random variable with known probability distribution. Shih (1980) used his models to investigate the impact of defective products in the lot size and the total cost of inventory system. Kalro and Gohil (1982) dealt

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with the problem to determine the economic lot size when the quantity of the lot received is uncertain due to various reasons such as rejections during inspection process, damage or breakage in transit. Their model also permits partial backorders. Perhaps the first work to relate quality and lot size was proposed by Porteus (1986). Porteus (1986) developed an EOQ inventory model that considers the effect of defective products and he assumed that there is a probability that the production system would go out of control. When the process is under control, good quality products are manufactured and when the process goes out of control, defective products are produced. Porteus's model (1986) suggests reducing the lot size in order to reduce the proportion of defective products generated by the manufacturing process. He also investigated the optimal setup investment in relation to reducing the probability of the process going out of control.

Rosenblatt and Lee (1986) studied the effects of an imperfect manufacturing process, which is assumed to deteriorate and produces some proportion of defective products on the optimal production cycle. They concluded that the presence of defective products generates smaller lot sizes than that of the classical EPQ inventory model. Schwaller (1988) extended the EOQ/EPQ inventory models by adding the assumption that a proportion of defective products in one lot is known. Schwaller's models consider two inspections costs: fixed and variable. Both are incurred in finding and removing the defective products. The Schwaller's EOQ inventory model also allows backorders, while Schwaller's EPQ inventory model does not. Zhang and Gerchak (1990) developed a model that considers a joint lot sizing and inspection policy with random yield where a random proportion of units are defective; and the defective products can not be used and must be replaced by good quality products. Lee, Chandra, and Deleveaux (1997) presented a mathematical model, based on EPQ inventory model, for obtaining the optimal economic production quantity and the proportion of defectives in a multi-stage manufacturing process environment with the restriction that all defective products must be scrapped. They also used their model for studying the effect of investment for quality improvement on the reduction of the proportion of defective products.

The presence of imperfect quality products in a lot size is another field that many researchers have been exploring recently. In this direction we have several works, for example, Salameh and Jaber (2000), extended the classical EOQ inventory model by accounting for imperfect quality products when using EOQ formulae. They considered that poor quality products are sold in a single lot at the end of the 100% screening process. Related to this work is the paper by Cárdenas-Barrón (2000) where an error appearing on Salameh and Jaber's model is corrected. The error is in the EOQ formulae but it does not diminish the main propose and contribution of the paper. Goyal and Cárdenas-Barrón (2002) presented a simple approach for determining economic production quantity for a product with imperfect quality. They also showed that nearly optimal results are obtained using this simple approach, which is much easier to implement than the optimal procedure proposed by Salameh and Jaber (2000). Goyal, Huang, and Chen (2003) extended the Goyal and Cárdenas-Barrón (2002) model to an integrated supplier-buyer inventory model that considers products of imperfect quality. They concluded firstly that there is an increment in the lot size when the value of the percentage of defective products increases. Secondly, there is a decrease in the number shipments per lot when the transportation cost increases. Thirdly, the number of shipments per lot, the lot size and the size of shipment increase as demand increases. Huang (2004) extended the Salameh and Jaber (2000) model to incorporate the view of the integrated single-vendor and single-buyer relationship, considering the presence of imperfect products into the lot size. On the other hand, Chang (2004) investigated the effects of products of imperfect quality by addressing two inventory models. The first one considers that the good quality products rate (defective rate) is represented by a fuzzy number and the demand is treated as constant and known. In the second inventory model, both the good quality rate (defective rate) and demand are represented by a fuzzy number. He concluded that his inventory models have the advantage of keeping uncertainties which capture real situations better than the Salameh and Jaber model (2000). All previous studies had considered that imperfect quality products are sold at a discounted price, but those studies do not address the impact of rejection and rework process and furthermore ignore the factor of when to sell. In this direction, Chan, Ibrahim, and Lochert (2003) developed a new EPQ inventory model that incorporates lower pricing, rework process and reject situations. They found that the time factor of when to sell imperfect quality products is very critical and does affect inventory cost and lot size. In a more recent paper, Papachristos and Konstantaras (2006) discussed that conditions proposed as sufficient to prevent shortages in the works of Salameh and Jaber (2000) and Chan et al. (2003) will not occur; those conditions can not guarantee this. Finally, Maddah and Jaber (2008) identified and rectified a flaw in the work of Salameh and Jaber (2000). Basically, Maddah and Jaber (2008) proposed a new model that rectifies the flaw using renewal theory. Other related research to the work of Salameh and Jaber (2000) include contributions by Eroglu and Ozdemir (2007), Wee, Yu, and Chen (2007) and Konstantaras, Goyal, and Papachristos (2007), to name a few.

The effect of the presence of defective products on lot size and rework of them has also received the attention of many researchers, including Gupta and Chakraborty (1984), Chakrabarty and Rao (1988), Lee (1992), Liu and Yang (1996), Richter and Sombrutzki (2000), Richter and Weber (2001), Minner and Kleber (2001), Hayek and Salameh (2001), Teunter and van der Laan (2002), Jamal, Sarker, and Mondal (2004) and Goyal and Cárdenas-Barrón (2005). A summary of the basic results of these studies is given below.

Gupta and Chakraborty (1984) developed an inventory model to determine both optimal production lot size and optimal rework lot size from the last stage to the first stage without considering the penalty cost of generating defective products. Chakrabarty and Rao (1988) have determined optimal lot size in a multi-stage production considering the rework of defective products. On the other hand, Liu and Yang (1996) considered a single-stage imperfect production system that may generate two types of defective products: reworkable products which should be sent back for remanufacturing and non-reworkable products which must be eliminated immediately. They showed that there is an optimal lot size and furthermore obtained a very efficient algorithm for calculating this optimal lot size. Lee (1992) dealt with the lot sizing problem in an imperfect manufacturing process. The features of imperfect manufacturing process that he considered were out of control detections, corrective actions to out of control states, rework setup times and variable rework times. Lee (1992) concluded that these features affect the optimal lot size in different ways. Richter and Sombrutzki (2000) and Richter and Weber (2001) combined the Wagner and Whitin (1958) algorithm and a pure reverse Wagner and Whitin model with given returns of used products. Richter and Weber (2001) included variable manufacturing and remanufacturing cost into the reverse Wagner and Whitin model. Minner and Kleber (2001) presented an optimal production and remanufacturing inventory model for a recovery model considering a linear cost functions. Hayek and Salameh (2001) studied the effect of imperfect quality products and rework of them on the finite economic production quantity model where shortages are allowed and backordered. They considered that the percentage of defective products follows a known probability density function. Teunter and van der Laan (2002) studied the reverse logistics discounted cash flow model with both remanufacturing and disposal of