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Economic production quantity model for randomly failing production process with minimal repair and imperfect maintenance

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ABSTRACT

This study applies periodic preventive maintenance (PM) to economic production quantity (EPQ) model for a randomly failing production process having a deteriorating production system with increasing hazard rate: minimal repaired and reworked upon failure (out of control state). The minimal repair performs restorations and returns the system to an operating state (in-control state). It is assumed that, after each PM, two types of PM are performed, namely imperfect PM and perfect PM. The probability that PM is perfect depends on the number of imperfect maintenance operations performed since the last renewal cycle. Mathematical formulas for the expected total cost are obtained. For the EPQ model, the optimum run time, required to minimize the total cost, is discussed. Various special cases are considered, including the maintenance learning effect. Finally, a numerical example is presented to illustrate the effect of PM and setup, breakdown and holding cost.

1. Introduction

To be globally competitive, manufacturers require a production policy that also manages inventory levels in the face of uncertainty regarding production failure and demand. Production policies have been extensively investigated with the aim of reducing production costs by Chelbi and Ait-Kadi (2004), Islam (2004), Jaber et al. (2009), Piñeyro and Viera (2010) and Ouyang et al. (2002). The economic production quantity (EPQ) model is a useful inventory control model that has been extensively investigated (Chung, and Huang, 2003). Notably, however, the quality of a product used to be inspected only at the final stage of production. Therefore, many non-conforming products have already been manufactured by the product inspection stage (Salameh and Jaber, 2000). Incorporating complete inspection into the production process has become possible with the automation of manufacturing. Hence, when the system shifts into "out-of-control" state, repair and reworking can be immediately performed to restore the state of the production process and satisfy product specifications or requirements.

Traditional EPQ models assume that a production process is free of quality issues. However, failure processes frequently occur in every workplace. It is more realistic to assume that production is sometimes imperfect. Such a production process is called imperfect production (Salameh and Jaber, 2000; Yoo et al., 2009). Rosenblatt and Lee (1986) examined the effect of process deterioration on optimal EPQ. Moreover, Porteus (1986) assumed that the probability of a shift from the "in-control" state to the "out-of-control" state has a given value for each production item. This investigation considers unexpected breakdown of production equipment to be inevitable. Following each failure, one of two breakdown policies is taken: (1) perform a major repair or (2) take a minimal repair, considering minimization of maintenance cost. Major repairs reset the system failure intensity (a good-as-new repair). Numerous protective systems, for example circuit breakers, alarms, and protective relays, are maintained in this fashion (Yang and Klutke, 2001). Because major repairs are expensive, minimal repairs are performed instead. The minimal repair does not alter the hazard rate of the system, but it restores the system to operational status. The idea of minimal repair was introduced in Barlow and Hunter (1960). Ja et al. (2001) illustrates that when a milling machine, drilling machine, or grinding machine breaks down, the broken machine is generally minimally repaired, and returned to its pre-failure state.

Preventative maintenance (PM) helps maintain a production system in top operating conditions. PM is crucial for complex systems because it reduces operating costs, and the risk of catastrophic failure. When a system is maintained at unequal intervals, the PM policy is known as sequential PM (Lin et al., 2000; Nakagawa, 1988). Another popular policy is the periodic PM policy, in which the system is preventively maintained at fixed time intervals (Chiang and Yuan, 2001; Nakagawa and Yasui, 1991).

Tseng (1996) introduced a perfect maintenance policy to increase the reliability of a deteriorating system. Perfect PM models assume that the system to be "as good as new" following maintenance. Some investigations based on maintenance and production analyses have sought to apply the imperfect production model to various real-world

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situations, including imperfect PM (Chung and Hou, 2003; Lin et al., 2003; Pham and Wang, 1996; Tseng et al., 1998). The models of imperfect PM assume that following PM the system lies in a state somewhere between as good as new and its pre-maintenance condition. The models of Nakagawa and Yasui (1987) and Nakagawa (1979) assumed that PM achieves either imperfect PM with a probability p, or perfect PM with probability $\overline{p}=1-p$. However, the assumption of imperfect PM is frequently false. Following PM, maintenance workers sometimes perform periodic testing for abnormalities. These tests help maintenance workers learn they cannot relax their attention and allow problems to recur. To include this learning experience, the probability that PM is perfect should depend on the number of imperfect maintenance operations performed since the last renewal cycle, and the probability that PM remains imperfect does not increase.

Few attempts have been made to integrate maintenance and production programs in a single model. Section 2 presents the integrate EPQ model. The model incorporates two possible restorations: periodic PM and minimal repair. That is, the PM is scheduled at cumulative production run time $jT(j=1,2,\ldots)$. The system involves minimal repair for response to failure exist. This study provides an integrate EPQ model for estimating the expected cost of performing these production, holding and restoration. The integrate EPQ model is determined, and the optimal production policy is thus determined.

Section 3 presents various special cases. Section 4 then provides numerical results for special cases. The study is conducted to investigate the effects of these parameters on the solution. Finally, Section 5 presents concluding remarks.

2. General model

To construct the model, relevant notations are defined as follows:

T	time of each production run
T_1	inventory depletion period; $T_1=((p/d)-1)T$
*	implies an optimum value
p	production rate in units per year
Q	production lot; $Q=p\cdot T$
d	demand rate in units per year; $p>d$
$\overline{P}_j$	probability that the first j PM are imperfect maintenances
p_j	probability that PM is perfect following the (j–1) imperfect PM; $p_j=\overline{P}_{j-1}-\overline{P}_j$
$\{\overline{P}_j\}$	sequence of $\overline{P}_j$, $j=0,1,2,\ldots$
q_j	Pr{a type-I PM when jth PM occurs}$=\overline{P}_j/\overline{P}_{j-1}$
θ_j	Pr{a type-II PM when jth PM occurs}$=1-q_j$
M	number of PM preceding the first type-II PM
C_m	cost of each PM
C_s	setup cost for each production run
C_{ms}	sum of C_m and C_s; $C_{ms}=C_m+C_s$
C_r	cost of each breakdown; minimal repair cost and rework cost
C_h	holding cost per unit per year of the product
$TC(T;\{\overline{P}_j\})$	expected total cost for EPQ model
$F(t)$	failure distribution function
$f(t)$	failure density function associated with $F(t)$
$\overline{F}(t)$	survival function associated with $F(t)$
$r(t)$	Hazard rate of a unit
Y	cumulative production run time between two successive renewal processes
r	The learning rate

In practice, determining production interval is a constant problem. The cost implications of EPQ model, such as those presented in this section, are important. This section considers a generalized EPQ model with minimal repair and PM using the following scheme. A system is maintained through two types of PM, that is, following periodic PM, the system may be either (1) unchanged or (2) renewed. The type I PM is termed imperfect PM, while the type II PM is known as the perfect PM. The probability of a type II PM depends on the number of PM occurring since the last renewal cycle. Let M denote the number of PM until the occurrence of the first type II PM. Additionally, let $\overline{P}_j=P(M>j)$. That is, $\overline{P}_j$ represents the probability that the first j outcomes are type I PM. Based on Sheu et al. (2005, 2006), this study assumes that the domain of $\overline{P}_j$ is {0,1,2,… }, and that $\overline{P}_0=1$. The probability $\overline{P}_j$ does not increase with the number of PM items j; $\overline{P}_{j-1}\geq\overline{P}_j(j=0,1,2,\ldots)$. This study uses the abbreviation $\{\overline{P}_j\}$ to represent a sequence of probabilities. Moreover, let $p_j=P(M=j)=\overline{P}_{j-1}-\overline{P}_j=\overline{P}_{j-1}(1-\overline{P}_j/\overline{P}_{j-1})$, with domain {1,2,3,…}. Consequently, when the jth PM occurs, then that PM is either type I with probability $q_j=\overline{P}_j/\overline{P}_{j-1}$, or type II with probability $\theta_j=1-q_j$.

To model the problem, the inventory cycle is divided into two major periods: inventory building period (production run period) and inventory depletion period. T and T_1 denote the time for inventory building and inventory depletion periods. Fig. 1 represents the inventory cycle of the EPQ model.

The system undergoes only minimal repair at failures between PM. Moreover, the following assumptions are made:

(1) The demand rate, production rate, setup cost and holding cost are known constants.
(2) Back order is not permitted.
(3) The original system begins operating at time 0. The production process starts in an in-control state and perfect items are produced.
(4) At the start of each inventory cycle, the setup cost C_s will be incurred. The cycle time for each production lot is T. PM is performed after production run period. The cost of each PM is C_m.
(5) A system has two types of PM at cumulative production run time $j\cdot T$ $(j=1,2,\ldots)$, based on outcome:
   - type-I PM (imperfect PM) results in the unit having the same failure rate as before PM, with probability $q_j=\overline{P}_j/\overline{P}_{j-1}(0\leq q_j<1)$;
   - type-II PM (perfect PM) makes the unit as good as new, with probability $\theta_j=1-q_j$.
(6) Following a perfect PM, the system returns to age 0.

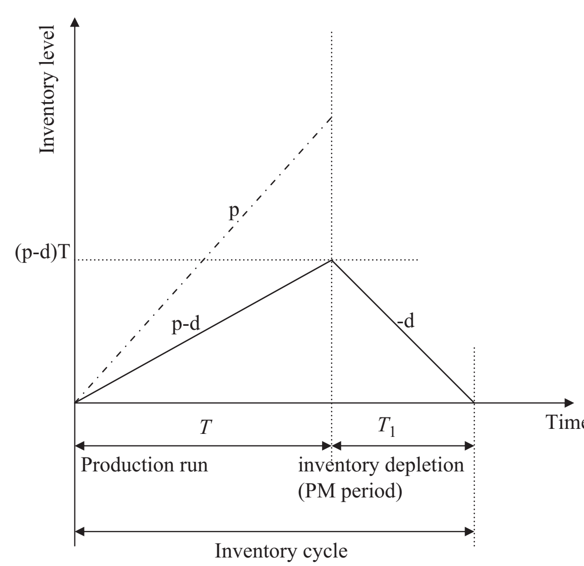

Fig. 1. The inventory cycle of the EPQ model with inventory building and depletion period.