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The economic production quantity with rework process in supply chain management

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ABSTRACT

Cardenas-Barron [L.E. Cardenas-Barron, Economic production quantity with rework process at a single-stage manufacturing system with planned backorders, Computers and Industrial Engineering 57 (2009) 1105–1113] minimizes the annual total relevant cost $TC(Q, B)$ to find the economic production quantity with rework process at a manufacturing system and assumes that $TC(Q, B)$ is convex. So, the solution $(\bar{Q}, \bar{B})$ satisfying the first-order-derivative condition for $TC(Q, B)$ will be the optimal solution. However, this paper indicates that $(\bar{Q}, \bar{B})$ does not necessarily exist although $TC(Q, B)$ is convex. Consequently, the main purpose of this paper is two-fold:

(A) This paper tries to develop the sufficient and necessary condition for the existence of the solution $(\bar{Q}, \bar{B})$ satisfying the-first-derivative condition of $TC(Q, B)$.
(B) This paper tries to present a concrete solution procedure to find the optimal solution of $TC(Q, B)$.

1. Introduction

Cardenas-Barron [1] minimizes the annual total relevant function $TC(Q, B)$ to find the economic production quantity with rework process at a manufacturing system with planned backorders and assumes that the annual total relevant cost $TC(Q, B)$ is convex. So, the solution $(\bar{Q}, \bar{B})$ satisfying the-first-derivative condition for $TC(Q, B)$ will be the optimal solution. However, this paper indicates that $(\bar{Q}, \bar{B})$ does not necessarily exist although $TC(Q, B)$ is convex. Consequently, the main purpose of this paper is two-fold:

(A) This paper tries to develop the sufficient and necessary condition for the existence of the solution $(\bar{Q}, \bar{B})$ satisfying the-first-derivative condition of $TC(Q, B)$.
(B) This paper tries to present a concrete solution procedure to find the optimal solution of $TC(Q, B)$.

2. The model

The model makes the following assumptions and notations that are used throughout this paper:
Assumptions:

(1) demand rate is constant and known over horizon planning;
(2) production rate is constant and known over horizon planning;
(3) the production rate is greater than demand rate;
(4) the production of defective products is known;

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Notations

D	Demand rate, units per time
P	Production rate, units per time ($P > D$)
R	Proportion of defective products in each cycle $\left(0 < R < 1 - \frac{D}{P}\right)$
K	Cost of a production setup (fixed cost), \$ per setup
C	Manufacturing cost of a product, \$ per unit
H	Inventory carrying cost per product per unit of time, $H = iC$
i	Inventory carrying cost rate, a percentage
W	Backorder cost per product per unit of time (linear backorder cost)
F	Backorder cost per product (fixed backorder cost)
Q	Batch size (units)
B	Size of backorders (units)
A	$1 - R$
E	$1 - R - \frac{D}{P}$
L	$1 - (1 + R + R^2)\frac{D}{P}$
T	Time between production runs
$TC(Q, B)$	Total cost per unit of time
Q^*, B^*	The optimal solution of $TC(Q, B)$.

(5) the products are 100% screened and the screening cost is not considered;
(6) all defective products are reworked and converted into good quality products;
(7) scrap is not generated at any cycle;
(8) inventory holding costs are based on the average inventory;
(9) backorders are allowed and all backorders are satisfied;
(10) production and reworking are done in the same manufacturing system at the same production rate;
(11) two types of backorder costs are considered: linear backorder cost (backorder cost is applied to average backorders) and fixed backorder cost (backorder cost is applied to maximum backorder level allowed);
(12) inventory storage space and the availability of capital is unlimited;
(13) the model is for only one product;
(14) the planning horizon is infinite.

Based on the above assumptions and notation, Cardenas-Barron [1] show that the total cost per unit of time $TC(Q, B)$ can be written as:

$$TC(Q, B) = \frac{KD}{Q} + \frac{HQL}{2} + \frac{HB^2A}{2QE} - HB + \frac{FBD}{Q} + \frac{WB^2A}{2QE} + CD(1 + R). \tag{1}$$

Eq. (1) shows that the respective partial derivatives with respect to Q and B can be expressed as:

$$\frac{\partial TC(Q, B)}{\partial Q} = -\frac{KD}{Q^2} + \frac{HL}{2} - \frac{HB^2A}{2Q^2E} - \frac{FBD}{Q^2} - \frac{WB^2A}{2Q^2E}, \tag{2}$$

$$\frac{\partial TC(Q, B)}{\partial B} = \frac{HBA}{QE} - H + \frac{FD}{Q} - \frac{WBA}{QE}. \tag{3}$$

Consider the first-order-derivative condition for $TC(Q, B)$

$$\frac{\partial TC(Q, B)}{\partial Q} = 0 \tag{4}$$

and

$$\frac{\partial TC(Q, B)}{\partial B} = 0. \tag{5}$$

Eqs. (4) and (5) imply

$$H\left[AL(H + W) - EH\right]Q^2 = 2KDA(H + W) - E(FD)^2, \tag{6}$$

$$A(H + W)B = E(HQ - FD). \tag{7}$$

3. The sufficient and necessary condition for the existence of the solution of the simultaneous Eqs. (4) and (5)

Let $(\bar{Q}, \bar{B})$ denote the solution of the simultaneous Eqs. (4) and (5).