The economic production quantity with rework process in supply chain management

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**ABSTRACT**

Cardenas-Barron [L.E. Cardenas-Barron, Economic production quantity with rework process at a single-stage manufacturing system with planned backorders, Computers and Industrial Engineering 57 (2009) 1105–1113] minimizes the annual total relevant cost \( TC(Q, B) \) to find the economic production quantity with rework process at a manufacturing system and assumes that \( TC(Q, B) \) is convex. So, the solution \((\bar{Q}, \bar{B})\) satisfying the first-order-derivative condition for \( TC(Q, B) \) will be the optimal solution. However, this paper indicates that \((\bar{Q}, \bar{B})\) does not necessarily exist although \( TC(Q, B) \) is convex. Consequently, the main purpose of this paper is two-fold:

(A) This paper tries to develop the sufficient and necessary condition for the existence of the solution \((\bar{Q}, \bar{B})\) satisfying the first-derivative condition of \( TC(Q, B) \).
(B) This paper tries to present a concrete solution procedure to find the optimal solution of \( TC(Q, B) \).

**1. Introduction**

Cardenas-Barron [1] minimizes the annual total relevant function \( TC(Q, B) \) to find the economic production quantity with rework process at a manufacturing system with planned backorders and assumes that the annual total relevant cost \( TC(Q, B) \) is convex. So, the solution \((\bar{Q}, \bar{B})\) satisfying the first-order-derivative condition for \( TC(Q, B) \) will be the optimal solution. However, this paper indicates that \((\bar{Q}, \bar{B})\) does not necessarily exist although \( TC(Q, B) \) is convex. Consequently, the main purpose of this paper is two-fold:

(A) This paper tries to develop the sufficient and necessary condition for the existence of the solution \((\bar{Q}, \bar{B})\) satisfying the first-derivative condition of \( TC(Q, B) \).
(B) This paper tries to present a concrete solution procedure to find the optimal solution of \( TC(Q, B) \).

**2. The model**

The model makes the following assumptions and notations that are used throughout this paper:

**Assumptions:**
1. Demand rate is constant and known over horizon planning;
2. Production rate is constant and known over horizon planning;
3. The production rate is greater than demand rate;
4. The production of defective products is known;

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Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>D</td>
<td>Demand rate, units per time</td>
</tr>
<tr>
<td>P</td>
<td>Production rate, units per time ((P &gt; D))</td>
</tr>
<tr>
<td>R</td>
<td>Proportion of defective products in each cycle ((0 &lt; R &lt; 1 - \frac{D}{P}))</td>
</tr>
<tr>
<td>K</td>
<td>Cost of a production setup (fixed cost), $ per setup</td>
</tr>
<tr>
<td>C</td>
<td>Manufacturing cost of a product, $ per unit</td>
</tr>
<tr>
<td>H</td>
<td>Inventory carrying cost per product per unit of time, (H = iC)</td>
</tr>
<tr>
<td>i</td>
<td>Inventory carrying cost rate, a percentage</td>
</tr>
<tr>
<td>W</td>
<td>Backorder cost per product per unit of time (linear backorder cost)</td>
</tr>
<tr>
<td>F</td>
<td>Backorder cost per product (fixed backorder cost)</td>
</tr>
<tr>
<td>Q</td>
<td>Batch size (units)</td>
</tr>
<tr>
<td>B</td>
<td>Size of backorders (units)</td>
</tr>
<tr>
<td>A</td>
<td>(1 - R)</td>
</tr>
<tr>
<td>E</td>
<td>(1 - R - \frac{P}{D})</td>
</tr>
<tr>
<td>L</td>
<td>(1 - (1 + R + R^2)\frac{D}{P})</td>
</tr>
<tr>
<td>T</td>
<td>Time between production runs</td>
</tr>
<tr>
<td>TC(Q, B)</td>
<td>Total cost per unit of time</td>
</tr>
<tr>
<td>Q*, B*</td>
<td>The optimal solution of (TC(Q, B)).</td>
</tr>
</tbody>
</table>

The products are 100% screened and the screening cost is not considered;
all defective products are reworked and converted into good quality products;
scrap is not generated at any cycle;
inventory holding costs are based on the average inventory;
backorders are allowed and all backorders are satisfied;
production and reworking are done in the same manufacturing system at the same production rate;
two types of backorder costs are considered: linear backorder cost (backorder cost is applied to average backorders) and fixed backorder cost (backorder cost is applied to maximum backorder level allowed);
inventory storage space and the availability of capital is unlimited;
the model is for only one product;
the planning horizon is infinite.

Based on the above assumptions and notation, Cardenas-Barron [1] show that the total cost per unit of time \(TC(Q, B)\) can be written as:

\[
TC(Q, B) = \frac{KD}{Q} + \frac{HQL}{2} + \frac{HB^2A}{2QE} - HB + \frac{FBD}{Q} + \frac{WB^2A}{2QE} + CD(1 + R). \tag{1}
\]

Eq. (1) shows that the respective partial derivatives with respect to \(Q\) and \(B\) can be expressed as:

\[
\frac{\partial TC(Q, B)}{\partial Q} = -\frac{KD}{Q^2} + \frac{HL}{2} - \frac{HB^2A}{2Q^2E} - \frac{FBD}{Q^2} - \frac{WB^2A}{2Q^2E}, \tag{2}
\]

\[
\frac{\partial TC(Q, B)}{\partial B} = \frac{HBA}{QE} - H + \frac{FD}{Q} - \frac{WBA}{QE}. \tag{3}
\]

Consider the first-order-derivative condition for \(TC(Q, B)\)

\[
\frac{\partial TC(Q, B)}{\partial Q} = 0 \tag{4}
\]

and

\[
\frac{\partial TC(Q, B)}{\partial B} = 0. \tag{5}
\]

Eqs. (4) and (5) imply

\[
H [AL(H + W) - EH] Q^2 = 2KDA(H + W) - E(FD)^2, \tag{6}
\]

\[
A(H + W)B = E(HQ - FD). \tag{7}
\]

3. The sufficient and necessary condition for the existence of the solution of the simultaneous Eqs. (4) and (5)

Let \((\tilde{Q}, \tilde{B})\) denote the solution of the simultaneous Eqs. (4) and (5).
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