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An economic production quantity model with a positive resetup point under random demand

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ABSTRACT

In this paper, an extended economic production quantity (EPQ) model is investigated, where demand follows a random process. This study is motivated by an industrial case for precision machine assembly in the machinery industry. Both a positive resetup point s and a fixed lot size Q are implemented in this production control policy. To cope with random demand, a resetup point, i.e., the lowest inventory level to start the production, is adapted to minimize stock shortage during the replenishment cycle. The considered cost includes setup cost, inventory carrying cost, and shortage cost, where shortage may occur at the production stage and/or at the end of one replenishment cycle. Under some mild conditions, the expected cost per unit time can be shown to be convex with respect to decision parameters s and Q. Further computational study has demonstrated that the proposed model outperforms the classical EPQ when demand is random. In particular, a positive resetup point contributes to a significant portion of this cost savings when compared with that in the classical lot sizing policy.

1. Introduction

Economic production quantity (EPQ) model is an extension of the well-known economic order quantity (EOQ) model where demand is assumed to be constant, e.g., see Hadley and Whitin [1]. Each replenishment cycle in EPQ consists of two stages: the duration to produce a fixed lot size where production rate is continuous and finite, and the duration to deplete remaining inventory before a new batch is launched. When the remaining inventory drops to zero, a new replenishment cycle will resume. An analytical model is developed to determine the optimal EPQ so that the sum of setup cost and inventory carrying cost is minimized. In practice, demand from customers can be dynamic or random. Donaldson [2] established an optimal replenishment schedule when demand is a linear function. Later, Deb and Chaudhuri [3] extended this work to allow for stock shortage. Hariga [4] developed a heuristic to find a lot sizing policy with linear demand when shortage is not permitted. An optimal replenishment schedule with linear or exponential time-varying demand has been studied by Hariga and Benkherouf [5], where an iterative numerical procedure is developed to find optimal schedule. The demand has been treated as deterministic with various functions of time in [2–5].

Economic order quantity models with imperfect quality and quantity discount can be found in Lin [6], where both vendor and buyer are considered. Chang [7] derived the exact expression for inventory carrying cost to improve this model, where simple and close-form formulas for computing the optimal order quantity are found. The optimal lot size and the maximum backorder level in an EOQ/EPQ model with fixed and linear backorder cost are studied in Cárdenas-Barrón [8]. Later, Cárdenas-Barrón [9] used a simple formula to determine the optimal number of shipments in Chang's [7] model. Sufficient and

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necessary conditions of the optimal replenishment policy and backorder level can be found in Chung and Cárdenas-Barrón [10]. The above studies have focused on the EOQ/EPQ policies for deterministic and constant demand process.

When faced with the random demand, inventory control policies with a positive reorder or resetup point s are the most often studied and adopted. For examples, the well known (reorder point s, fixed lot size Q) and (reorder point s, order-up-to-level S) policies, where the reorder point provides safety stock to cope with demand uncertainty. It has been shown that the control policies with reorder point outperform the classical EOQ or EPQ policy, where the reorder point for procurement or resetup point for production is $s = 0$, e.g., see Hadley and Whitin [1]. Rabinowitz et al. [11] studied an economic order quantity model when demand follows a Poisson distribution and shortage is treated as partial backorder. Namit and Chen [12] analyzed an (s, Q) inventory system when demand is Gamma distributed. Simulation is used to find optimal replenishment quantity that minimizes expected cost per unit time.

In the EPQ context, Gavish and Graves [13] considered a production lot sizing problem with Poisson demand, where a (resetup point s, production up-to-level S) policy has been studied. The production continues until stock level reaches S; and production is resumed when stock level falls to or below s in this control policy. Note that the lot size varies from one replenishment cycle (current batch) to the next cycle. More recently, Rempala [14] studied a production lot sizing policy for a compound Poisson demand process, where partial backorder and bi-level inventory control are considered. These analyses have focused on theoretical aspects of the control policy. There are only a few papers that incorporate both a resetup point and a fixed lot size in the production control policies, e.g., see Groenevelt et al. [15] and Palar [16], due to the intractability of the analytical models and computational difficulty to find the optimal policies.

Another stream of production policies has focused on the effects of imperfect production process. Rosenblatt and Lee [17] first developed an economic production cycle model that deals with deteriorating production process and defective items. An EPQ model in two-stage production system is analyzed in Lee and Rong [18], where both mean time to failure and mean time to repair are random variables. Later, Goyal and Giri [19] studied an optimal lot sizing model for a deteriorating item with time-varying demand. The optimal production cycle time for an unreliable production system with rework is considered in Chiu et al. [20]. Konstantaras et al. [21] developed an adapted lot sizing model when the learning effect on imperfect quality of the production process and shortage are considered. These studies have focused on production policies for deterministic and known demand process.

This research was motivated by a case study for the precision machine assembly of an international supplier. These precision machines are used to, for an example, fabricate turbine blades for the aircraft or parts for high-precision products. The setup cost is extremely high in the assembly system. An order from customers usually consists of one unit for a specific model of machine; and average annual orders can be reasonable estimated. The time between orders, however, is unpredictable. The shortage cost is difficult to quantify and shortage is not allowed for this industry. Due to random demand, a shortage may occur during the production period and/or at the end of the replenishment cycle. For a shortage that occurred during the production period, it is backordered and immediately filled with the finished product from the assembly line. This is defined as a transient shortage. For a shortage occurred at the end of the replenishment cycle, it will be backordered and satisfied with new stock. The later is defined as a cycle shortage. In addition, due to high production cost, fierce competition, and rapid market change, both excessive inventory and stock shortage are to be avoided. This production lot sizing problem is modeled and analyzed with a control policy that includes a positive resetup point s and a fixed lot size EPQ, i.e., (s, Q) policy.

In the following sections, we first formulate this extended lot sizing problem when a production system faces random demand, then derive and analyze the expected cost model. Section 3 gives an efficient search procedure to find the optimal resetup point, the production lot size, and its minimal cost value, based on the analytic results. Numerical experiments are conducted to gain understanding of the extended production policy; and some parametric analyses are reported. A brief concluding summary is given in the last section.

2. Problem description and model formulation

An extended production lot sizing policy with a positive resetup point s and a fixed lot size Q is formulated and analyzed in this section. The development of the analytical model is a culmination of a case study for the assembly of precision machines. Most of the times, the order received from customers is one unit, due to high procurement cost of machines. Time between orders exhibits no pattern, and is infrequent and difficult to observe, although the average orders per year can be reasonably estimated. Due to the uniqueness of precision machining industry, shortage is prohibited. A resetup point s is used to cope with such a stock shortage. A fixed production quantity Q is also implemented in the system, which serves as the most commonly found policy for operation and scheduling in production systems, enabling the minimization of workload fluctuation in production lines and the simplification of planning and control.

We consider a production system that processes one item at a time with constant production rate μ. Demand of customers follows a stationary Poisson process with rate λ, that is, $P\{Q_d(0,t) = k\} = e^{-\lambda t}(\lambda t)^k/k!,\ k = 0, 1, \ldots$, where $\mu > \lambda > 0$, and $Q_d(0, t)$ denotes total demand from time 0 to t. Each replenishment cycle can be partitioned into two stages: duration of production period $T_1 = [0, Q/\mu)$, where lot size Q is processed with constant production rate μ, and duration of depletion period $T_2 = [Q/\mu, \sim)$, where remaining inventory at time Q/μ is used to meet future demand. A replenishment cycle starts when the inventory level falls to or below s and the production system is idle, where $I_i(t),\ i = 1, 2$ denotes the inventory level at time t