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An economic production quantity model with consolidating shipments of imperfect quality items: A note

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ABSTRACT

In this note, we correct some typos that appeared in Yassine et al. {Yassine, A., Maddah, B., Salameh M., Disaggregation and consolidation of imperfect quality shipments in an extended EPQ model, International Journal of Production Economics 135 (2012) 345–352}, specifically, for one of their models that considered consolidating shipments of imperfect quality items across multiple production cycles. In addition, we present a heuristic approach to find a good solution for this model. The performance of heuristic solution is illustrated with numerical examples.

1. Introduction

In order to capture the real situation, numerous studies have modified the traditional economic order/production quantity (EOQ/EPQ) models to include imperfect quality items, and proposed different ways to handle them. For a detailed review and discussion of the related literature, we refer the readers to Khan et al. (2011). For up-to-date research, see e.g. Rezaei and Salimi (2012), Hsu and Hsu (2012), etc.

Recently, Yassine et al. (2012) extended the EPQ models to address the issues of shipping the imperfect quality items through disaggregating (into smaller batches within a single production run) or consolidation (across multiple production runs) of imperfect quality items. We have read the paper with a considerable interest. However, we found some typos in their consolidating shipments model, where one of the decision variables (namely the number of production cycles (n_c) between two consecutive shipments of imperfect quality items) is determined through solving a cubic equation. We also found some typos in the numerical results. This note rectifies the above mentioned results and presents an easy-to-compute heuristic solution for n_c. Numerical experiments are carried out to illustrate the performance of this solution.

2. Clarify Yassine et al.'s equations

In the consolidating shipments model of Yassine et al. (2012, p. 349), the equations needing clarification are as follows.

The holding cost of imperfect quality items over a shipping cycle composed of n_c production cycles was given by

$$CI_h(y_c,n_c)=h\left\{\sum_{i=1}^{n_c-1}\left[\frac{y_c^2}{2\alpha}P_i+\frac{y_c^2}{\beta}\left(1-P_i-\frac{\beta}{\alpha}\right)P_i\right]+\sum_{i=1}^{n_c-2}y_cP_i\sum_{j=i+1}^{n_c-1}\frac{y_c}{\beta}(1-P_j)+\sum_{j=1}^{n_c-1}\frac{y_c^2}{2\alpha}P_j+\frac{y_c^2}{2\alpha}P_{n_c}\right\} \quad (1)$$

In (1), the third term inside the brace represents the inventory from cycle i carried to production cycle n_c, which should be $\sum_{j=1}^{n_c-1}(y_c^2/\alpha)P_j$ (the sum of rectangular areas) rather than $\sum_{j=1}^{n_c-1}(y_c^2/2\alpha)P_j$ (the sum of triangular areas).

By taking the expected value of (1), they derived

$$E[CI_h(y_c,n_c)]=(n_c-1)y_c^2\left(\frac{1}{\beta}-\frac{1}{2\alpha}\right)\mu-\frac{(n_c-1)y_c^2}{\beta}(\mu^2+\sigma^2)+\frac{(n_c-1)(n_c-2)y_c^2}{2\beta}(\mu^2-\mu)+\frac{(n_c-1)y_c^2}{\alpha}\mu+\frac{y_c^2}{2\alpha}\mu \quad (2)$$

In (2), the third term should be $((n_c-1)(n_c-2)y_c^2/2\beta)(\mu-\mu^2)$, and all the terms should be multiplied by 'h' as shown in (1).

By simplifying (2), they derived

$$E[CI_h(y_c,n_c)]=\frac{hy_c^2}{2\beta}n_c\left[n_c\mu(1-\mu)+\frac{\beta}{\alpha}\mu-2\sigma^2\frac{n_c-1}{n_c}\right] \quad (3)$$

In (3), the first term inside the bracket should be $(n_c-1)\mu(1-\mu)$. Note that when $n_c=1$, (3) becomes $(hy_c^2\mu(1-\mu)/2\beta)+(hy_c^2\mu/2\alpha)$, but the expected accumulation of imperfect quality items is $E[(1/2)(y_c/\alpha)P_1y_c]=(y_c^2\mu/2\alpha)$.

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The expected holding cost of imperfect quality items per unit time was given by

$$E[CI_{hu}(y_c,n_c)]=\frac{E[CI_h(y_c,n_c)]}{E(SC)}=\frac{hy_c}{2(1-\mu)}\left[n_c\mu(1-\mu)+\frac{\beta}{\alpha}\mu-2\sigma^2\frac{n_c-1}{n_c}\right] \tag{4}$$

As in (3), the first term inside the bracket in (4) should be $(n_c-1)\mu(1-\mu)$.

Finally, they derived the expected total cost.

$$ETCU_c(y_c,n_c)=\frac{1}{(1-\mu)}\left\{\frac{(K_s/n_c+K)\beta}{y_c}+\frac{hy_c}{2}\left[(1-\mu)^2-\frac{\beta}{\alpha}(1-2\mu)+(n_c-1)(\mu-\mu^2)+\left(\frac{2}{n_c}-1\right)\sigma^2\right]\right\} \tag{5}$$

It should be pointed out that using the corrected results we derived the expected total cost as shown in (5). Thus, those errors could be typos, while they may confuse the readers without clarification.

3. A heuristic solution

In order to present a heuristic solution, we first briefly review Yassine et al.'s approach. For fixed n_c, they derived the optimal production quantity and the minimum value of $ETCU_c(n_c,y_c)$:

$$y_c^*=\sqrt{\frac{2(K+(K_s/n_c))\beta}{\theta_2 h}}, \tag{6}$$

where $\theta_2=(1-\mu)^2-(\beta/\alpha)(1-2\mu)+(n_c-1)(\mu-\mu^2)+((2/n_c)-1)\sigma^2$ and

$$ETCU_c(n_c)=ETCU_c(n_c,y_c^*(n_c))=\sqrt{2g_2(n_c)}/(1-\mu), \tag{7}$$

where $g_2(n_c)=(K+(K_s/n_c))\beta h\theta_2$.

From (7), they further derived a cubic equation for solving the optimal continuous value of n_c (say $\tilde{n}_c$):

$$n_c^3+\gamma n_c+\pi=0, \tag{8}$$

where $\gamma=\{[-2\mu^2+3\mu+(\beta/\alpha)(1-2\mu)+\sigma^2-1](K_s/K)-2\sigma^2\}/(\mu-\mu^2)$ and $\pi=-4(K_s/K)\sigma^2/(\mu-\mu^2)$.

Finally, the optimal integer value of n_c was obtained by

$$n_c^*=\arg\min_{\lfloor\tilde{n}_c\rfloor,\lceil\tilde{n}_c\rceil}(ETCU_c(n_c)) \tag{9}$$

where $\lfloor x\rfloor$ is the largest integer $\le x$ and $\lceil x\rceil$ is the smallest integer $\ge x$.

From (9), we see that n_c^* is a rounding value of $\tilde{n}_c$ while $\tilde{n}_c$ cannot be obtained without a numerical search method. For ease of computation, we present a heuristic solution (say $\hat{n}_c$) to replace $\tilde{n}_c$. Observing π in (8), where $\sigma^2/(\mu-\mu^2)$ is small in general (for instance, $P\sim U(0,b)$, if $b=0.2$, then $\mu=b/2=0.1$, $\sigma^2=b^2/12=0.000833$, $\sigma^2/(\mu-\mu^2)=b/(6-3b)=0.037$) and $K_s/K<1$ is often considered, it can be conjectured that the value of π is small. Ignoring π in (8) yields

$$\hat{n}_c=\sqrt{\frac{1}{\mu(1-\mu)}\left\{\frac{K_s}{K}\left[(1-\mu)(1-2\mu)-\sigma^2-\frac{\beta(1-2\mu)}{\alpha}\right]+2\sigma^2\right\}} \tag{10}$$

if $\hat{n}_c^*=1$ then $\hat{n}_c^*=1$, otherwise $\hat{n}_c^*=\arg\min_{\lfloor\hat{n}_c\rfloor,\lceil\hat{n}_c\rceil}ETCU_c(n_c)$.

When $n_c=1$, (5) and (6) respectively reduce to

$$ETCU_c(y_c)=\frac{(K+K_s)\beta}{(1-\mu)y_c}+\frac{hy_c}{2(1-\mu)}\left[(1-\mu)^2+\sigma^2-\frac{\beta(1-2\mu)}{\alpha}\right]\equiv ETCU(y_b) \tag{11}$$

and

$$y_c^*=\sqrt{\frac{2\beta(K+K_s)}{h\{(1-\mu)^2+\sigma^2-[\beta(1-2\mu)/\alpha]\}}}\equiv y_b^* \tag{12}$$

Note that in Yassine et al.'s (2012) numerical study, to make the base model comparable with the disaggregating and consolidating shipments models, they modified their original base model to include shipment setup cost K_s (i.e., (11) and (12) were adopted).

4. Numerical results

We first correct the typos appeared in Yassine et al.'s numerical results, which were obtained using the data: $\alpha=100{,}000$, $\beta=50{,}000$, $K=100$, $K_s=50$, $h=5$, and $P\sim U(0,\ b)$ with $b=0.04$ and $b=0.15$. For the case $b=0.04$, they obtained $y_b^*=2499$ with $ETCU(y_b^*)=6003$ for the base model (the same for the disaggregating shipments model with $n_d^*=1$); however, the exact total cost obtained from (5) (or (11)) should be $ETCU(y_b^*)=3062.9+3062.9=6125.8$. For the consolidating shipments model, they obtained $y_c^*=2043$, $n_c^*=4$ with $ETCU_c(y_c^*,n_c^*)=5507$. Solving the cubic equation (8), we obtained $\tilde{n}_c=3.43063$. But, for $\lfloor\tilde{n}_c\rfloor=3$ and $\lceil\tilde{n}_c\rceil=4$, since $ETCU_c(3)=5617.6<ETCU_c(4)=5619.3$, the exact optimal value of n_c should be $n_c^*=3$. To confirm this solution, we list the results obtained by (5) and (6) for given n_c=2, 3, 4. From Table 1, it is clear that $n_c^*=3$, $y_c^*=2119.2$, and $ETCU_c(y_c^*,n_c^*)=$ 5617.6. Moreover, by the closed-form heuristic (10), $\hat{n}_c=3.43006$ follows. Thus, for this case, we can conclude that $\hat{n}_c^*=\arg\min_{\lfloor\hat{n}_c\rfloor,\lceil\hat{n}_c\rceil}ETCU_c(n_c)=3=n_c^*$.

Next, we demonstrate the performance of proposed heuristic approach. Let us work on the cases recommending 'aggregate' (i.e., consolidation) model listed in Table 2 (case #1–5) and Table 3 (case #1–2 and case #7–9) of Yassine et al. (2012, pp.350-351). The computing results are summarized in Table 2.

From Table 2, we see that $\hat{n}_c-\tilde{n}_c<0$ (i.e., $\hat{n}_c$ is underestimated) and the largest error is 0.0103 (resulting from the largest omitted value $|\pi|=0.1081$), and hence for all cases the heuristic solution $\hat{n}_c^*$ reaches optimum value. Once $\hat{n}_c^*=n_c^*$ is confirmed, the values of y_c^* and $ETCU_c(y_c^*,n_c^*)$ should be the same as their values, while there are slight differences, which could be due to the effects of rounding.

In addition, we set up 20 testing cases, taking $\beta=10{,}000$ (a low demand rate), $K_s=500$ (a higher shipment cost than setup cost K), and $b=0.05$ (0.05) 0.5. The computing results are summarized in Table 3.

Again, the results in Table 3 show $\hat{n}_c-\tilde{n}_c<0$. The largest error ($=0.1251$) occurs at the bottom case ($K_s/K=5$ and $b=0.5$) in which the heuristic optimal $\hat{n}_c^*=3$ is found by comparing the total costs of $\lfloor\hat{n}_c\rfloor=2$ and $\lceil\hat{n}_c\rceil=3$, while the exact optimal $n_c^*=3$ is found by comparing $\lfloor\tilde{n}_c\rfloor=3$ and $\lceil\tilde{n}_c\rceil=4$. In our numerical experiments so far, we always have $\hat{n}_c^*=n_c^*$. Thus, to find an optimal solution for n_c, using the simplified solution $\hat{n}_c$ is more efficient than using $\tilde{n}_c$.

5. Conclusion

This note corrected some typos that appeared in Yassine et al.'s (2012) consolidating shipments model. It also provided a heuristic solution with closed-form expression for determining the near-optimal number of production cycles. Numerical results indicate no optimality gap of the heuristic in the tested cases.

Table 1
The results of (5) and (6) for n_c=2, 3, 4.

n_c	θ_2	y_c	$ETCU_c(y_c,n_c)$
2	0.5000	2236.1	5704.3
3	0.5196	2119.2	5617.6
4	0.5391	2042.9	5619.3