Economic production quantity models for deteriorating/imperfect products and service with rework ☆

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ABSTRACT

In this paper, two economic production quantity (EPQ) models are proposed for deteriorating/imperfect items with rework process. The production process is imperfect, and imperfect quality items are reworked to become serviceable. At the same time, the remaining good quality items may deteriorate. The inspection of deteriorated items is also imperfect, so that deteriorated items may be sold to customers, which will create negative impact on corporate image. In the first model, a single production-rework plant system is considered. The optimal production times and the economic production quantities are obtained analytically. In the second model, a system that consists of n production plants and one rework plant is considered. A solution procedure is developed in order to obtain the optimal operating cost. Numerical examples are provided to compare the performance of the two systems.

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1. Introduction

Serving high quality products and providing good service can always attract customers and keep them coming back. However, in reality, production processes are often imperfect. For economic and environmental reasons, imperfect quality items are reworked to become serviceable again. Due to unsuitable inventory condition or other reasons,the remaining good quality items stored in inventory face deterioration. In order to provide good service to customers, inspection is carried out to screen out deteriorated items. However, such inspection may not be perfect and only part of deteriorated items can be screened out. The remaining deteriorated items will then be sold to customers. It will lower customer satisfaction and is harmful to corporate image (Abratt & Mofokeng, 2001; Smith & Taylor, 2004).

Foodstuffs, volatile liquids such as gasoline and alcohol, radioactive substances and pharmaceutical drugs are common deteriorating items considered in the literature; see (Wee, 1993). One possible industry that this research work can be applied to is soft-drink filling industry. An item produced is a bottle filled by soft-drink and it is classified as imperfect quality if the amount of liquid filled is out of a specified range (Tai, Ching, & Chan, 2009). The rework process may be referred to refilling the bottle.

In this paper, we consider two models in which the products produced are imperfect and the service provided to customers is also imperfect. The products are imperfect in two ways. Firstly, during the production process, imperfect quality items may be produced. Inspection, which occurs immediately after production, is used to identify imperfect items which are sent to rework and good quality items which are stocked and made available for sale to customers. Secondly, items may deteriorate in the inventory. Hence, inspection processes for deteriorated items are carried out among the stored items in the inventory. The service is imperfect in two ways. Firstly, inspections for deteriorated items in inventory are assumed to be imperfect. One common source of inspection error is from human factors; see (Drury, 1978; Drury & Prabhu, 1994). Deteriorated items may be sold to customers,which will lead to customer dissatisfaction. Secondly, shortage is allowed and unsatisfied demands are backlogged. Since not all customers accept late delivery, partial backlogging is also considered in the paper.

In the first model, we consider a single production plant system. The plant is also capable of rework processing. Part of the imperfect quality items are recovered and ready to be sold to customers. At the beginning of the production process, the amount of backlog is made up. The production process continues until the total number of items produced reaches the economic production quantity Q^*. When production ends the rework process starts immediately until all imperfect quality items are processed. Demands are then satisfied by the serviceable items in the inventory. After the inventory level becomes zero, further demands are backlogged and satisfied at the beginning of the next cycle.

In the second model, we consider a system that consists of a central rework plant, which is capable of handling imperfect quality items for rework only, and n local production plants. The

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behavior of the local production plants is the same of the previous model except they cannot handle rework process. After the production processes in the local plants, the imperfect quality items are aggregated and shipped to the central plant for rework. The imperfect quality items are then recovered and the recovered items are for satisfying the demands at the central plant. Due to the difference between the inventory models of the rework plant and the production plants, substantial cost savings can be realized. The remainder of this paper is organized as follows. In Section 2, we give a literature review and the motivation of this study. In Section 3, we consider a model for the a single production plant with rework capability. In Section 4, we consider an aggregated model for a central rework plant and n local production plants. We then give numerical examples in Section 5. Finally, concluding remarks are given in Section 6.

2. Literature review

Economic production quantity (EPQ) is one of the main research topics in production and inventory management. By using EPQ model, optimal quantity of items produced can be obtained. Classical EPQ model was developed under various assumptions. Since then, researchers have extended the model by relaxing one or more of its assumptions.

One basic assumption in classical EPQ model is that the items produced are of perfect quality. However, imperfect quality items may be produced in reality. Salameh and Jaber (2000) proposed an EPQ model with imperfect quality products. The defective items are screened out and sold as a single batch at a lower price. The paper was then revisited by Papachristos and Konstantaras (2006), in which the defective items are withdrawn at the end of a production cycle. Vörös (2012) considered an EPQ model with varying defective rates between cycles. Rezaei and Davoodi (2008) considered a supply chain with multiple products and multiple suppliers. Received items from suppliers were not of perfect quality and decision was made by using genetic algorithm. Chung, Her, and Lin (2009) proposed an inventory model with two warehouses, where one of them was rented. Yassine, Maddah, and Salameh (2012) considered disaggregating the shipments of imperfect quality items in a single production run and aggregating the shipments of imperfect items over multiple production runs. Hsu and Hsu (2013) considered the case when there is inspection error in the screening process of imperfect quality items. A review on EPQ models for imperfect quality items can be found in (Khan, Jaber, Guiffrida, & Zolfaghari, 2011).

How to handle imperfect quality items is another important issue. One possible way is to perform rework and make them become serviceable. Chan, Ibrahim, and Lochert (2003) provided an EPQ model which considered lower pricing, rework and reject situations. The items produced were classified as good items, good items after rework, imperfect quality items and rejected items. They assumed that the quality of an item was quantifiable and normal distributed. Jamal, Sarker, and Sanjay Mondal (2004) proposed a model to obtain the optimal batch quality in a single-stage production system. Rework was done under two operational policies such that total system cost was minimized. Chiu, Wang, and Chiu (2007) determined the optimal run time for an EPQ model with scrap, rework, and stochastic machine breakdowns. Buscher and Lindner (2007) considered a two-stage manufacturing system in which production and rework activities were carried out. The economic production and rework quantity and the corresponding batch sizes were determined. Taleizadeh, Sadjadi, and Niaki (2011) studied two joint production systems in a form of multi-product single machine with and without rework. Yoo, Kim, and

Park (2009) proposed an EPQ model that incorporated both imperfect production quality and two-way imperfect inspection. Glock and Jaber (2013) considered a multi-stage production-inventory system and measured the performance by four partial measures based on production time, process yield, in-process inventory and shipment frequency.

Goods are considered as deteriorating items because their values go down with time. Products such as electronic products, fashion clothing, food and chemical are common examples. Teng and Chang (2005) proposed an EPQ model for deteriorating items with the demand rate dependant on the selling price of the products and the stock level. Lin, Kroll, and Lin (2006) considered the economic lot scheduling problem (ELSP) for deteriorating items. The problem was to schedule multiple products to be manufactured on a single machine repetitively over an infinite planning horizon. Liao (2007) developed a production model with finite production rate and considered the effect of deterioration and permissible delay in payments. Chung and Wee (2011) considered short life-cycle deteriorating items with green product design. Widyadana and Wee (2012) proposed an EPQ model for deteriorating items with rework, which was preformed after m production setups.

Another key assumption of classical EPQ model is that no shortage is allowed. Shortage may be handled in two ways: backorders and lost sales. Wee, Yu, and Wang (2006) proposed an integrated model for deteriorating items in which shortages were completely backordered. A periodic delivery policy for a vendor and a production-inventory model for a buyer were established. Later, Chang and Ho (2010) used the renewal-reward theorem to derive the exact closed-form solutions of the optimal lot size, backordering quantity and maximum expected net profit per unit time. Cárdenas-Barrón (2009) extended the models in (Jamal et al., 2004) by considering planned backorders. Partial backordering is considered if lost sales is allowed. Mak (1987) proposed a optimal production-inventory policy for an inventory system with partial backordering. Pentico, Drake, and Toews (2009) investigated the model in (Mak, 1987) and redeveloped it for the EPQ with partial backordering using simpler expressions. Wee (1993) proposed an economic production policy for deteriorating items with partial backordering using iterative method. Giri, Jalan, and Chaudhuri (2005) considered an EPQ model with increasing demand rate and adjustable production rate while shortage are partial backlogged. Teng, Ouyang, and Chen (2007) gave a comparison between two pricing and lot-sizing models with partial backlogging and deteriorated items.

We notice that not many studies considered a model with imperfect quality and deteriorating items, rework and shortage. On the other hand, the effect of selling deteriorated items to customers has not been addressed fully either. In this paper, we aim at providing analytic models which are absent in the literature. In the proposed models, the following characteristics are considered: (i) imperfect and deteriorating items; (ii) inspection error, (iii) rework and (iv) shortage. We remark that most, if not all, models in the literature considered part of the above characteristics only.

3. The basic models

In this section, we consider a single production plant system which is also capable of handling imperfect quality items produced during the production process. The following notations are used throughout the paper.

p production rate (unit/unit time)
α percentage of good quality items produced
λ demand rate (unit/unit time)