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A new fuzzy dempster MCDM method and its application in supplier selection

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ABSTRACT

Supplier selection is a multi-criterion decision making problem under uncertain environments. Hence, it is reasonable to hand the problem in *fuzzy sets theory* (FST) and *Dempster Shafer theory* of evidence (DST). In this paper, a new MCDM methodology, using FST and DST, based on the main idea of the technique for order preference by similarity to an ideal solution (TOPSIS), is developed to deal with supplier selection problem. The *basic probability assignments* (BPA) can be determined by the distance to the ideal solution and the distance to the negative ideal solution. Dempster combination rule is used to combine all the criterion data to get the final scores of the alternatives in the systems. The final decision results can be drawn through the pignistic probability transformation. In traditional fuzzy TOPSIS method, the quantitative performance of criterion, such as crisp numbers, should be transformed into fuzzy numbers. The proposed method is more flexible due to the reason that the BPA can be determined without the transformation step in traditional fuzzy TOPSIS method. The performance of criterion can be represented as crisp number or fuzzy number according to the real situation in our proposed method. The numerical example about supplier selection is used to illustrate the efficiency of the proposed method.

1. Introduction

Many decision-making applications, such as supplier selection, within the real world inevitably include the consideration of evidence based on several criteria, rather than on a preferred single criterion. A lot of researchers have devoted themselves to solve multi-criteria decision-making (MCDM) (Bouyssou, 1986; Gal & Hanne, 2006; Narasimhan & Vickery, 1988; Shyur & Shih, 2006; Wadhwa, Madaan, & Chan, 2009). Due to the flexibility to deal with uncertain information, it is necessary to use fuzzy sets theory (FST) and Dempster Shafer theory of evidence (DST). Fruitful papers about MCDM based on FST (Ashtiani, Haghighirad, & Makui, 2009; Chu & Lin, 2009; Deng & Liu, 2005a, 2005b; Deng, 2006; Hu, 2009; Hanaoka & Kunadhamraks, 2009; Olson & Wu, 2006; Wu & Olson, 2008; Yang, Chiu, & Tzeng, 2008; Yeh & Chang, 2009; Zhang, Wu, & Olson, 2005) and DST are published (Bauer, 1997; Beynon, Curry, & Morgan, 2000, 2001; Beynon, 2002, 2005; Deng, Shi, & Liu, 2004; Mercier, Cron, & Denoeux, 2007; Srivastava & Liu, 2003; Wu, 2009; Yager, 2008; Yang & Sen, 1997; Yang & Xu, 2002).

Recently, Wu (2009) proposed a method to select international supplier using grey related analysis and Dempster–Shafer theory to deal with this fuzzy group decision making problem. Grey related analysis (Deng, 1982) is employed as a means to reflect uncertainty in multi-attribute models through interval numbers in the individual aggregation. The Dempster–Shafer combination rule is used to aggregate individual preferences into a collective preference in the group aggregation.

In this paper, however, we proposed another MCDM methodology using FST together with DST. The new method has some desired properties. First, the proposed method uses linguistic items modeled as fuzzy numbers to represents experts' subjective opinions in addition of crisp number to rank the performance of criterion. Whether using quantitative representation or qualitative representation is depending on the real situation. This property is very desired for multiple experts decision making since there are not only quantitative data but also qualitative representation in the process of decision making. Second, based on the DST, the subject fuzzy numbers can be easily combined with the crisp numbers. That is, the proposed method can efficiently fuse quantitative and qualitative data in a straightforward manner. Third, the proposed method can be easily implemented step by step to solve MCDM problems.

The remaining paper is organized as follows: Section 2 briefly introduce the preliminaries of fuzzy sets theory (FST) and DST. In Section 3, our fuzzy Dempster method to deal with MCDM is proposed. A numerical example to supplier selection is used to show the efficiency of the proposed method. Finally, some conclusions are made in Section 5.

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2. Preliminaries

In this section, we simply introduce some relative mathematics tools, such as fuzzy sets theory (FST) and Dempster Shafer theory of evidence (DST), which will be used in our new proposed method.

2.1. Fuzzy sets theory

2.1.1. Fuzzy number

Definition 2.1 (*Fuzzy set*). Let X be a universe of discourse, $\widetilde{A}$ is a fuzzy subset of X if for all $x \in X$, there is a number $\mu_{\widetilde{A}}(x) \in [0,1]$ assigned to represent the membership of x to $\widetilde{A}$, and $\mu_{\widetilde{A}}(x)$ is called the membership of $\widetilde{A}$ (Zimmermann, 1991).

Definition 2.2 (*Fuzzy number*). A fuzzy number $\widetilde{A}$ is a normal and convex fuzzy subset of X. Here, the "Normality" implies that (Zimmermann, 1991).

$$\exists x \in \mathbb{R}, \quad \bigvee_x \mu_{\widetilde{A}}(x) = 1 \tag{1}$$

and "Convex" means that

$$\begin{aligned} &\forall x_1 \in X, \quad x_2 \in X, \quad \forall \alpha \in [0,1], \\ &\mu_{\widetilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geqslant \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)) \end{aligned} \tag{2}$$

Definition 2.3. A triangular fuzzy number $\widetilde{A}$ can be defined by a triplet (a,b,c) shown in Fig. 1. The membership function is defined as Zimmermann (1991).

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leqslant x \leqslant b \\ \frac{c-x}{c-b}, & b \leqslant x \leqslant c \\ 0, & x > c \end{cases} \tag{3}$$

2.1.2. Linguistic variable

The concept of linguistic variable is very useful in dealing with situations which are too complex or ill-defined to be reasonably described in conventional quantitative expressions. Linguistic variables are represented in words or sentences or artificial languages, where each linguistic value can modeled by a fuzzy set (Kauffman & Gupta, 1985). In this paper, the importance weights of various criteria and the ratings of qualitative criteria are considered as linguistic variables. For example, these linguistic variables can be expressed in positive triangular fuzzy numbers as Table 1. It should be noticed that there are many different methods to represent linguistic items. Which kind of represent method is used is depend on the real application systems and the domain experts' opinions.

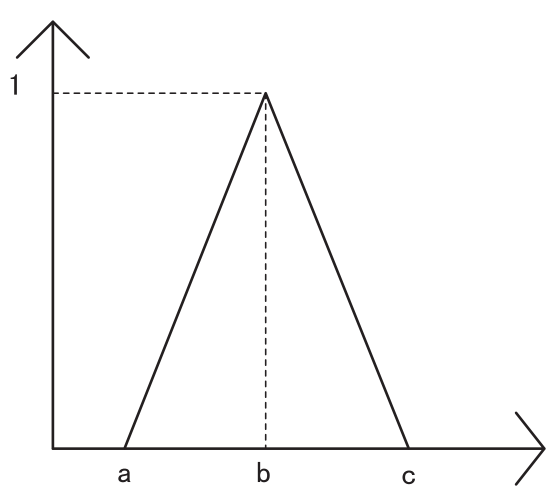

Fig. 1. A triangular fuzzy number.

Table 1
Linguistic variables for the importance weight and ratings.

Very low (VL)	(0,0.1,0.3)
Low (L)	(0.1,0.3,0.5)
Medium (M)	(0.3,0.5,0.7)
High (H)	(0.5,0.7,0.9)
Very high (VH)	(0.7,0.9,1.0)

2.1.3. Defuzzification

Defuzzification is an important step in fuzzy modeling and fuzzy multi-criteria decision-making. The defuzzification entails converting the fuzzy value into a crisp value, and determining the ordinal positions of n-fuzzy input parameters vector. Many defuzzification techniques are available (Zimmermann, 1991), but the common defuzzification methods include centre of area, first of maximums, last of maximums, and middle of maximums (MoM).

Different defuzzification techniques extract different levels of information. In this paper, the canonical representation of operation on triangular fuzzy numbers (Chou, 2003), which is based on the graded mean integration representation method is used in defuzziness process. For detailed information, please refer (Chou, 2003).

Definition 2.4. Given a triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$, the graded mean integration representation of triangular fuzzy number $\widetilde{A}$ is defined as

$$P\left(\widetilde{A}\right) = \frac{1}{6}(a_1 + 4 \times a_2 + a_3) \tag{4}$$

By applying Eq. (4), the graded mean integration representation for importance weight of each criterion and ratings are shown in Table 2

2.2. Dempster shafer theory of evidence

DST (Dempster, 1967; Shafer, 1976) can be regarded as a general extension of Bayesian theory that can robustly deal with incomplete data. In addition to this, DST offers a number of advantages, including the opportunity to assign measures of probability to focal elements, and allowing for the attachment of probability to the frame of discernment. In this section, we briefly review the basic concepts of evidence theory.

Evidence theory first supposes the definition of a set of hypotheses Θ called the frame of discernment, defined as follows:

$$\Theta = \{H_1, H_2, \ldots, H_N\}$$

It is composed of N exhaustive and exclusive hypotheses. Form the frame of discernment Θ, let us denote $P(\Theta)$, the power set composed with the 2^N propositions A of Θ:

$$P(\Theta) = \{\emptyset, \{H_1\}, \{H_2\}, \ldots, \{H_N\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\} \ldots, \Theta\}$$

where Ø denotes the empty set. The N subsets containing only one element are called singletons. A key point of evidence theory is the basic probability assignment (BPA). The mass of belief in an element of Θ is quite similar to a probability distribution, but differs by the fact that the unit mass is distributed among the elements of $P(\Theta)$,