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Exploitation of a single species by a threshold management policy

Magno Enrique Mendoza Meza [a,*], Michel Iskin da Silveira Costa [b,*]

[a] Federal University of ABC (UFABC), Center of Engineering, Modeling and Applied Social Sciences (CECS), Av. dos Estados 5001, SP09210-580, Bloco A, Sala 744-1, Bangu, Santo André, São Paulo, Brazil

[b] Lab. Nacional de Computação Científica, Av. Getúlio Vargas, 333 – Quitandinha, Petrópolis-RJ 25651-070, Brazil

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ABSTRACT

Continuous time models of single exploited populations usually generate outcomes expressing a dependence of yield and economic items on harvest intensity. In this work it is shown that a known threshold policy is able to generate yield and related economic items that do not depend on harvest intensity, but rather on the values of the population threshold itself and the species intrinsic parameters. It is argued that since this result can be carried over to other models of single species dynamics, it may have significant implications in the management and conservation of exploited populations.

1. Introduction

Single as well as multispecies harvesting has been a subject of theoretical and empirical studies [21]. As to single species exploitation, fixed quota and fixed proportion harvest policies have been applied in the management of natural populations both in continuous and discrete time settings. Besides the two strategies named above, threshold policies have also been proposed as a means of population management [19]. Basically, threshold policies consist of two or more thresholds that dictate different harvest strategies according to the population level with respect to the considered thresholds. In this work it is shown that threshold strategies of exploited populations can generate sustainable yield and related economic items with harvest intensities (e.g., fishing effort) that would otherwise cause species extinction if they were continuously applied. To some extent, this result can be carried over to other continuous single species models and therefore may have significant implications in the management of exploited populations regarding economic and conservation issues.

2. A threshold policy applied to a logistic growth model under a fixed proportion harvest strategy

A single population continuous time model under continuous harvest can be cast as follows:

$$\frac{dx}{dt} = f(x) - g(x). \tag{1}$$

$f(x)$ is the productivity as a function of species density x. A commonly used form of this function is the logistic growth, i.e., $f(x) = rx(1 - x/K)$, where r is the species intrinsic growth rate, K is the carrying capacity. $g(x)$ is a density dependent function dictating any rate per unit time of species removal (e.g., harvest, grazing, etc.). This function can take on many forms, such as: (i) $g(x) = \varepsilon x$ – a fixed proportion harvest rate per unit time; (ii) $g(x) = C$ – a constant harvest per unit time. Both instances can lead to species extinction, depending on their magnitude.

Given the general setting (1), the proposed threshold policy (hereinafter called TP), $\phi(x)$, applied to (1) yields

$$\frac{dx}{dt} = f(x) - \phi(x)g(x), \tag{2}$$

with

$$\phi(x) = \begin{cases} \alpha_1, & \text{if } x > x_{th}, \\ \alpha_2, & \text{if } x \leqslant x_{th}, \end{cases} \quad (\alpha_1, \alpha_2 \geqslant 0). \tag{3}$$

x_{th} is the threshold level that should be chosen according to the problem to be solved (see Fig. 1 for $x_{th} > 0$).

The model given by (2) and (3) is equivalent to

$$\frac{dx}{dt} = \begin{cases} f(x) - \alpha_1 g(x) & \text{if } x > x_{th}, \\ f(x) - \alpha_2 g(x) & \text{if } x \leqslant x_{th}. \end{cases} \tag{4}$$

$\alpha_1 > 0$ and $\alpha_2 = 0$ describe the important and frequently used case when harvest is completely curtailed at low population levels.

* Corresponding authors. Tel.: +55 11 4996 0100; fax: +55 11 4996 0089 (M.E.M. Meza), Tel.: +55 24 2233 6008; fax: +55 24 2233 6141 (M.I.S. Costa).

E-mail addresses: magno.meza@ufabc.edu.br (M.E.M. Meza), michel@lncc.br (M.I.S. Costa).

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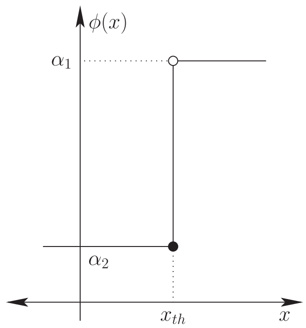

Fig. 1. Graphical representation of the TP with α_1, $\alpha_2 > 0$ and $x_{th} > 0$.

When α_1, $\alpha_2 > 0$ with $\alpha_2 < \alpha_1$, it represents the case where harvest intensity is reduced at low population levels.

Within this setting, this policy creates two systems (actually, two structures, and hence the name *variable structure system* [22]) with their own equilibrium points, separated by the threshold level. A schematic phase plane x, dx/dt is depicted in Fig. 2.

Notice in Fig. 2 that the choice of the threshold can be made so that the equilibrium points of each structure are placed in opposite regions.

Definition 1. If the equilibrium points are located in their opposite regions, they are named virtual equilibrium points. Otherwise, they are called real equilibrium points.

Hence in Fig. 2, if both equilibrium points are locally or globally stable and virtual, they will never be attained by their respective dynamics, since they change as soon as the trajectories cross the threshold.

Consider a logistic growth rate, $f(x) = rx(1 - \frac{1}{K})$, together with a proportional harvest rate, $g(x) = \varepsilon x$. Consider also $\alpha_1 = 1$ and $\alpha_2 = 0$, i.e., the removal (with intensity ε) is enacted whenever the population level x is above x_{th}, while removal is stopped when $x \leqslant x_{th}$. This yields,

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right) - \phi(x)\varepsilon x, \tag{5}$$

which is equivalent to

$$\frac{dx}{dt} = \begin{cases} rx\left(1 - \frac{x}{K}\right) - \varepsilon x & \text{if } x > x_{th}, \\ rx\left(1 - \frac{x}{K}\right) & \text{if } x \leqslant x_{th}. \end{cases} \tag{6}$$

System (5) or (6) consists of two structures: (i) no harvest region (NH) where $x \leqslant x_{th}$, with a globally stable equilibrium point $x_{NH} = K$; (ii) harvest region (H) where $x > x_{th}$ and $\varepsilon > 0$, which generates a globally stable equilibrium point $x_H = K(1 - \frac{\varepsilon}{r})$. For any $\varepsilon > 0$, $x_{NH} > x_H$. That is to say, if the no-harvest equilibrium point is in the no-harvest region, then the harvest equilibrium point must be in the same region.

Hence, when $\varepsilon > 0$ in the logistic model, the choice of a threshold such that $K(1 - \frac{\varepsilon}{r}) < x_{th} < K$, will render the globally stable equilibrium points of each structure – $x_H = K(1 - \frac{\varepsilon}{r})$ and $x_{NH} = K$ – virtual (located in opposite regions), and therefore they can never be attained by their respective dynamics. Besides, this position of the threshold x_{th} creates opposed vector fields (the signs of dx/dt of each structure) in its vicinity, as depicted in Fig. 3 (cf. the schematic view in Fig. 2).

However, it is important to stress that in the case of Fig. 3 (i.e., $\varepsilon < r$), the occurrence of a sliding mode depends on ε, since virtual equilibrium will exist only if $x_H = K(1 - \frac{\varepsilon}{r}) < x_{th} < K$.

Fig. 4(a) is a specific case of Fig. 3 where (without loss of generality) $\varepsilon \geqslant r$, which inevitably causes species extinction in the logistic model (i.e., $N_H = 0$). Accordingly, choosing $0 = x_H < x_{th} < x_{NH} = K$, both globally stable equilibrium points of each structure will be virtual. For any initial condition $x(0) > 0$ in any region, the trajectory will tend towards the opposite region (see the arrows in Fig. 4(a)). As soon as the trajectory crosses x_{th}, the vector field will send it back to the opposite region. This alternation of the signs of dx/dt in the neighborhood of the threshold will entail a sliding mode along x_{th} [22], as shown in the time dynamics of Fig. 4(b). So, in the variable structure system given by model (6), the conjunction of virtual equilibrium and opposing vector fields in the vicinity of the threshold can generate a sliding mode (however, it is important to mention that virtual equilibrium is not a necessary condition for a sliding mode to occur [8,9]). That is to say, the severely harvested species (under a fixed proportion harvest strategy) that would otherwise go extinct stabilizes at a new steady state (the chosen threshold x_{th}) by means of rapid alternation of harvest intensities $\varepsilon = 0$ and $\varepsilon \geqslant r$ along the species threshold x_{th}. It is important to emphasize, though, that stabilization of a heavily exploited species is attained at the cost of rapidly alternating

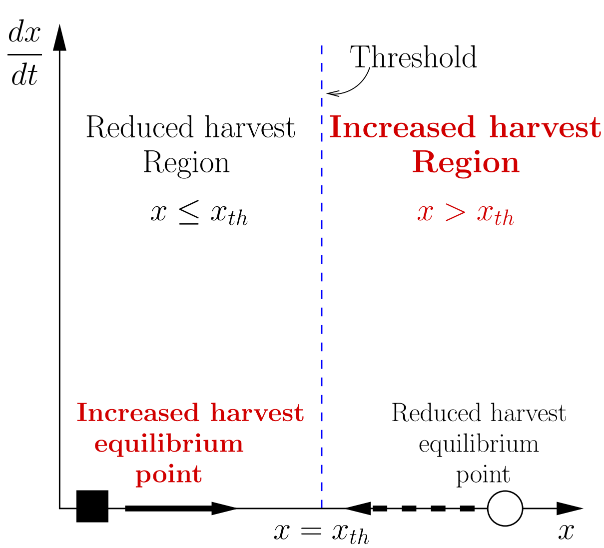

Fig. 2. An hypothetical plane x, dx/dt where the reduced harvest and reduced harvest regions are determined by the level $x = x_{th}$ (dashed line). Notice that the equilibrium points are located in opposite regions, hence they are virtual equilibrium points. ○ – equilibrium point of the reduced harvest region; ■ – equilibrium point of the increased harvest region. The solid arrow indicates the vector field directed to the no harvest equilibrium point, while the dashed arrow indicates the vector field directed to the harvest equilibrium point.

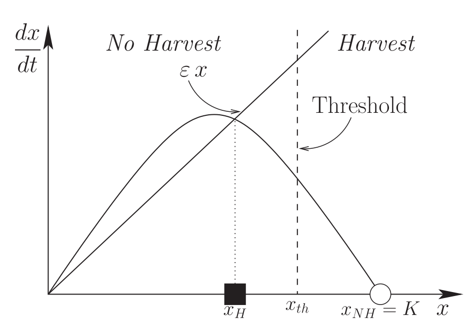

Fig. 3. The logistic growth curve, the harvesting rate curve (slanted line) and the stopping rule x_{th} (vertical line) depicted in the plane $x, \frac{dx}{dt}$. In this case, $x_H = K(1 - \frac{\varepsilon}{r})$, a globally stable equilibrium point for the harvest structure (■), lies in the no harvest region; in turn, $x_{NH} = K$ is also a globally stable point for the no harvest case (○) lies in the harvest region-both are virtual equilibrium points. Besides, the vector fields of each structure point to each other in the vicinity of x_{th}.