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Multifractality in the stock market: price increments versus waiting times

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Abstract

By applying the multifractal detrended fluctuation analysis to the high-frequency tick-by-tick data from Deutsche Börse both in the price and in the time domains, we investigate multifractal properties of the time series of logarithmic price increments and inter-trade intervals of time. We show that both quantities reveal multiscaling and that this result holds across different stocks. The origin of the multifractal character of the corresponding dynamics is, among others, the long-range correlations in price increments and in inter-trade time intervals as well as the non-Gaussian distributions of the fluctuations. Since the transaction-to-transaction price increments do not strongly depend on or are almost independent of the inter-trade waiting times, both can be sources of the observed multifractal behaviour of the fixed-delay returns and volatility. The results presented also allow one to evaluate the applicability of the Multifractal Model of Asset Returns in the case of tick-by-tick data.

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1. Introduction

From the perspective of the classical financial market models, behaviour of the consecutive price and index fluctuations does not present any significant time autocorrelations except for short time scales up to several minutes. As regards the dynamical character of these fluctuations, they are considered as being to a good approximation the fractional Gaussian noise [1], with very small and negligible probability of the occurrence of non-Gaussian large jumps in the index or the share price. As a natural consequence, the stock market data is expected to present only monofractal properties. However, these widely used models do not describe the processes underlying the evolution of financial data with satisfactory precision. The so-called financial stylized facts comprising, among others, the non-negligible fat tails of log-return distributions, volatility clustering and its long-time correlations, anomalous diffusion, etc. [2–5] counter the above-mentioned fundamental assumptions of market dynamics challenging their applicability in practice. That the financial dynamics is more complex than it is commonly assumed can also be inferred from a number of recently published papers discovering and exploring the multifractal characteristics of data from the stock markets [6–10], the forex markets [7,10–13] and the commodity ones [14]. The concept of multifractality was developed in order to describe the scaling properties of singular measures and functions which exhibit the presence of various distinct scaling exponents in their different parts [19,20]. Soon the related formalism was successfully applied to characterize empirical data in many distant fields like turbulence [21,22], earth science [23], genetics [24–26], physiology [27–29] and, as already mentioned, in finance. The problem of detecting multifractality in real data is delicate, however. There are models based on fractal processes which are able to mimic the real multifractal evolution of markets being either multifractal or monofractal [15,18,30–32]. Moreover, as it has been pointed out [18,33], the power of commonly used tests of multifractality is limited, because they cannot effectively distinguish between the two types of fractal behaviour of the financial (but perhaps also other) data. One important source of this difficulty is the presence of non-Gaussian tails in the distributions of data (e.g. truncated Lévy [33]), the fact which is ubiquitous in finance. Thus, all conclusions drawn from multifractal analysis have to be interpreted with care.

In the present paper we analyze data from the German stock market focusing on their fractal properties. We apply the multifractal detrended fluctuation analysis which is a well-established method of detecting scaling behaviour of signals. By exploiting the character of the high-frequency transaction-by-transaction recordings for the most liquid stocks belonging to DAX, we are able to inspect not only the properties of share price fluctuations, but also the properties of time intervals between consecutive trades (waiting times). The majority of analyses carried out so far was devoted to time series of the returns calculated with some fixed time delay Δt. According to the Multifractal Model of Asset Returns introduced by Mandelbrot and others [15–18,31], the source of multifractality in the returns is a deformation of time $\theta(t)$, which takes place due to the fact that at the microscale the so-called business time is "paced" by transactions rather than any constant time units. In this