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The longer-horizon predictability of German stock market volatility☆

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Abstract

This paper examines the predictability of the volatility of the German DAX stock index over a range of 1–45 trading days with a new test procedure. In contrast to earlier findings, according to which the volatility of the DAX is only predictable about 15 trading days into the future, the new test suggests that it is predictable about 40 trading days ahead. Moreover, predictability does not decay too quickly with horizon. An out-of-sample forecasting experiment with alternative volatility models supports the new empirical evidence.

JEL classification: C12; C53; G10

Keywords: Stock market volatility; Volatility forecasting; Predictability; Interval forecast evaluation; Density forecast evaluation

1. Introduction

Many empirical studies find that the volatility (i.e., the variance or standard deviation) of financial returns is predictable over short forecasting horizons, such as one day or one week (Poon & Granger, 2003 provide a survey). Less is known about the predictability of volatility over longer horizons. However, financial institutions need longer term volatility forecasts as well, for pricing and hedging financial derivatives, for example. Volatility forecasts also play an important role in model-based calculations of the market risk inherent in the trading book of a bank. Regulators usually require a ten day horizon in such calculations. Economic policy makers also look at financial market volatility. For instance, the European Central Bank routinely examines longer term volatility forecasts of important financial series in its Monthly Bulletin.

Some studies (e.g., Brailsford & Faff, 1996; Figlewski, 1997; Pagan & Schwert, 1990) evaluate the forecasting performance of certain volatility models over weekly, monthly, and sometimes even longer horizons. Such model-based evaluations are of course informative, but problematic if one is interested in the predictability of volatility per se because the results may not only vary with the forecasting horizon, but also with the assumed models. To avoid

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this joint hypothesis problem Christoffersen and Diebold (2000) use a "model-free" test procedure to examine the predictability of the volatility of a number of financial series. They also ask how strong the predictability at different horizons might be. They found that volatility is largely unpredictable beyond ten to twenty trading days. Moreover, they find that the strength of predictability declines quickly.

In this paper we develop an alternative model-free test procedure for assessing the predictability of volatility. The test combines the definition of unpredictability given in Clements and Hendry (1998) with results from the density forecast evaluation literature (surveyed in Clements, 2005; and Tay & Wallis, 2000). A Monte Carlo experiment indicates that the new test is more powerful in the case of fat-tailed conditional return distributions than the runs test of Christoffersen and Diebold (2000) or the classical ARCH test of Engle (1982) which may also be interpreted as a test for the predictability of volatility. Under conditional normality the test is about as powerful as the ARCH test but more powerful than the runs test. The three tests are then applied to the German DAX stock index, an important index which has not been extensively studied in the volatility forecasting literature. The runs test and the ARCH test find essentially no volatility predictability beyond 15–20 trading days. In contrast, the new test suggests that the volatility of the DAX is predictable over a range of approximately 40 trading days. A simple measure of the strength of predictability indicates that predictability does not decay too quickly with horizon.

The empirical results obtained with the new test suggest that more sophisticated approaches such as, for example, GARCH-type models, should help to forecast the DAX volatility over longer horizons. To see whether this is the case we conduct an out-of-sample experiment where the volatility of the DAX is forecasted 10, 30 and 45 trading days ahead. In the experiment, GARCH-based volatility forecasts and implied volatility are indeed more accurate than historical volatility (an estimate of the unconditional variance).

The paper proceeds as follows. Section 2 introduces the notion of predictability used in this paper. Section 3 describes the new test procedure, Section 4 reports Monte Carlo results and Section 5 presents the empirical findings about the predictability of the DAX volatility. The out-of-sample forecasting experiment and its results are described in Section 6. The last section contains some concluding remarks.

2. Predictability

Clements and Hendry (1998) define a random variable y_t to be unpredictable with respect to an information set Ω_{t-1} if the conditional distribution $F_{y_t}(y_t|\Omega_{t-1})$ and the unconditional distribution $F_{y_t}(y_t)$ of y_t coincide, i.e. if

$$F_{y_t}(y_t|\Omega_{t-1}) = F_{y_t}(y_t). \tag{1}$$

This notion of unpredictability states that the information set Ω_{t-1} does not improve the prediction of y_t. If Ω_{t-1} is restricted to past realizations of y_t, then Eq. (1) implies that past realizations do not help to predict y_t.[1]

Now consider a sample of "centered" returns $\{e_t=r_t-E(r_t|\Omega_{t-1})\}_{t=1}^{j}$ where the conditional mean dynamics in the raw returns r_t have been removed. The joint distribution of the centered returns can be factored into the product of $j-1$ conditional distributions and a marginal distribution

$$F(e_j, e_{j-1}, \ldots, e_1) = \prod_{t=1}^{j} F(e_t|\Omega_{t-1}). \tag{2}$$

Assuming that the e_ts have the same marginal distribution F, the conditional variance of the centered returns is unpredictable if $F(e_t|\Omega_{t-1})=F(e_t)$ holds for each e_t, that is

$$F(e_j, e_{j-1}, \ldots, e_1) = \prod_{t=1}^{j} F(e_t). \tag{3a}$$

Hence, the data should be independently and identically distributed (i.i.d.) with distribution function F if volatility cannot be predicted from past returns.

[1] Unpredictability is defined relative to an information set. Therefore, unpredictability of y_t with respect to Ω_{t-1} does not imply that y_t could not be predicted with the help of another information set $\Psi_{t-1}\neq\Omega_{t-1}$. The volatility of a return series could for example be unpredictable from past returns but predictable from quoted option prices.