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# Long memory in stock market volatility and the volatility-in-mean effect: The FIEGARCH-M Model

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ABSTRACT

We extend the fractionally integrated exponential GARCH (FIEGARCH) model for daily stock return data with long memory in return volatility of Bollerslev and Mikkelsen (1996) by introducing a possible volatility-in-mean effect. To avoid that the long memory property of volatility carries over to returns, we consider a filtered FIEGARCH-in-mean (FIEGARCH-M) effect in the return equation. The filtering of the volatility-in-mean component thus allows the co-existence of long memory in volatility and short memory in returns. We present an application to the daily CRSP value-weighted cum-dividend stock index return series from 1926 through 2006 which documents the empirical relevance of our model. The volatility-in-mean effect is significant, and the FIEGARCH-M model outperforms the original FIEGARCH model and alternative GARCH-type specifications according to standard criteria.


## 1. Introduction

Many of the salient features of daily stock returns are well described by the FIEGARCH (fractionally integrated exponential generalized autoregressive conditional heteroskedasticity) model introduced by Bollerslev and Mikkelsen (1996). Thus, in addition to time-varying volatility and volatility clustering (the ARCH and GARCH effects, as in Engle (1982) and Bollerslev (1986)), and the resulting unconditional excess kurtosis or heavier than normal tails, the model accounts for long memory in volatility (fractional integration, as in the FIGARCH model of Baillie et al. (1996)), as well as asymmetric volatility reaction to positive and negative return innovations (the exponential feature, as in Nelson's (1991) EGARCH model).

In this paper, we introduce a filtered in-mean generalization of the FIEGARCH model, which we label FIEGARCH-M. The generalization allows a volatility feedback or risk-return relation effect of changing conditional volatility on conditional expected stock returns, and generates unconditional skewness. Following recent literature (Ang et al., 2006, and Christensen and Nielsen, 2007), it is changes in volatility that enter the return equation. The filtering of volatility when entering it in the return specification implies that the long memory property of volatility (the fractionally integrated feature) does not spill over into returns, which would be empirically unrealistic.

That volatility exhibits long memory is well established in the recent empirical literature. This finding is consistent across a number of studies[1], and financial theory may accommodate long memory in volatility as well, see Comte and Renault (1998).

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[1] See, e.g., Robinson (1991), Crato and de Lima (1994), Baillie et al. (1996), Ding and Granger (1996), Breidt et al. (1998), Robinson (2001), and Andersen et al. (2003).

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Many of the studies use GARCH-type frameworks, but none of them consider in-mean specifications, i.e., parametric relations across conditional means and variances[2]. The FIEGARCH-M model of the present paper fills this gap.

Three related effects may introduce a relation between volatility and mean returns, namely, (i) a risk-return tradeoff capturing the risk premium required by investors as compensation for taking on additional risk, (ii) a financial leverage effect, and (iii) a volatility feedback effect. We briefly discuss each of these in turn.

Early theoretical and empirical contributions on the risk-return relation were due to Merton (1973, 1980). In equilibrium, investors taking on additional risk should be compensated through higher expected return, which implies a positive coefficient in the risk-return relation. The GARCH-M (GARCH-in-mean) model proposed by Engle et al. (1987) allows for the direct effect of volatility changes on asset prices through required returns in a short memory GARCH-type model, by introducing the conditional volatility function into the conditional mean return equation. Empirical studies of the risk-return tradeoff using GARCH-type models for stock returns obtain mixed results regarding both the sign and the significance of the in-mean effect, see e.g. Bollerslev et al. (1988), Chou (1988), Glosten et al. (1993), Nelson (1991), Campbell and Hentschel (1992), and Chou et al. (1992). Recent work in asset pricing examines cross-sectional risk premia induced by covariance between innovations in volatility and stock returns. This literature finds negative premia, e.g. Ang et al. (2006). The idea is that since innovations in volatility are higher during recessions, stocks which co-vary with volatility are stocks that pay off in bad states, and these should require a smaller risk premium. For a survey of related studies, see Lettau and Ludvigson (2004).

While time-varying volatility in itself generates excess kurtosis in unconditional distributions, which is common to most financial return series, the phenomenon that negative return innovations induce higher volatility than positive innovations of the same magnitude, observed particularly in stock return distributions, may be accommodated using the EGARCH model of Nelson (1991). The asymmetric volatility reaction pattern may stem from a financial leverage effect, see e.g. Black (1976), Engle and Ng (1993), and Yu (2005). The standard argument from Black (1976) is that bad news decrease the stock price, hence increasing the debt-to-equity ratio (i.e. financial leverage), and equity carries all asset risk, making the stock relatively riskier after the price drop and increasing future expected volatility.

An alternative source of a negative volatility-return relation is the volatility feedback mechanism of Campbell and Hentschel (1992), that is, if volatility is increased, then so is the risk premium, in case of a positive tradeoff between risk and conditional expected return. Hence, the discount rate is also increased, which in turn for an unchanged dividend yield lowers the stock price. Presumably, the volatility feedback effect should be strongest at the market level, whereas the leverage effect should apply to individual stocks.

Our FIEGARCH-M model includes both the exponential (asymmetry) and in-mean features, thus allowing tests of whether both are empirically relevant. Although the causality is reversed, the leverage and volatility feedback effects may be seen as supplementing each other as explanations of the negative return-volatility relation documented in empirical stock market research. In the empirical model, the negative relation may show up both through the exponential and the in-mean feature. Of these, only the latter generates unconditional skewness (see He et al., 2008). It is worth noting that the volatility feedback mechanism induces a negative volatility-return relation even in the presence of a positive equity premium or risk-return tradeoff, and for a given data frequency the negative feedback effect may dominate the positive tradeoff effect in the estimation of the in-mean volatility-return relation. At the relatively high, say daily, frequencies where GARCH-style models are most useful, the initial price reaction through the change in discount rate (the feedback mechanism) is relatively more important than the change in mean return (asset pricing or tradeoff) effect of a volatility change, and so the estimated in-mean effect may to a larger extent reflect feedback. Our model allows estimating both the exponential and volatility-in-mean effects simultaneously, and the estimated in-mean volatility-return relation will point to a feedback or tradeoff effect operating alongside the leverage effect.

We apply our FIEGARCH-M model to the CRSP value-weighted cum-dividend stock index return series using daily data from 1926.1.2 through 2006.12.29. We estimate the model by quasi-maximum likelihood (QML). The validity of the robust (sandwich-formula) standard errors is confirmed using the wild bootstrap algorithm. We compare the model to a number of alternative GARCH-type specifications, including standard GARCH, IGARCH, Spline-GARCH, FIGARCH, Adaptive FIGARCH, EGARCH, FIEGARCH, and associated models with in-mean effects, such as GARCH-M. The comparison confirms that FIEGARCH is preferred over other models without in-mean features. Furthermore, in-mean features in fact further improve the fit. The best model according to standard information criteria rewarding both goodness-of-fit and parsimony as well as to out-of-sample forecasting performance is the new FIEGARCH-M specification. In particular, the volatility-in-mean effect is statistically significant, even when controlling for autocorrelation in daily returns. Thus, the results demonstrate that the volatility-in-mean effect indeed is an empirically important extension of the original FIEGARCH model.

In the next section, we present our FIEGARCH-M model, which incorporates all the above mentioned features. Section 3 presents the application to the daily CRSP data, and Section 4 concludes.

## 2. The FIEGARCH-M model

We extend the FIEGARCH model by introducing volatility into the return equation, i.e., the in-mean feature, along the lines of the GARCH-M literature, thus yielding a new FIEGARCH-M model. Since long memory in volatility introduced into the return

[2] To the best of our knowledge, the only study of the relation between volatility with long memory and conditional mean returns is Christensen and Nielsen (2007), which is outside the GARCH-class, using instead a stochastic volatility model and basing inference on realized (from high-frequency returns) volatility or implied (from option prices) volatility.