Journal of Empirical Finance 18 (2011) 147–159

Contents lists available at ScienceDirect

Journal of Empirical Finance

journal homepage: www.elsevier.com/locate/econbase

Multivariate fractionally integrated APARCH modeling of stock market volatility: A multi-country study☆

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A R T I C L E I N F O

Available online 10 May 2010

JEL classification:
C13
C22
C32
C52
C53

Keywords:
Asymmetric power ARCH
Fractional integration
Stock returns
Volatility forecast evaluation

A B S T R A C T

Tse (1998) proposes a model which combines the fractionally integrated GARCH formulation of Baillie, Bollerslev and Mikkelsen (1996) with the asymmetric power ARCH specification of Ding, Granger and Engle (1993). This paper analyzes the applicability of a multivariate constant conditional correlation version of the model to national stock market returns for eight countries. We find this multivariate specification to be generally applicable once power, leverage and long-memory effects are taken into consideration. In addition, we find that both the optimal fractional differencing parameter and power transformation are remarkably similar across countries. Out-of-sample evidence for the superior forecasting ability of the multivariate FIAPARCH framework is provided in terms of forecast error statistics and tests for equal forecast accuracy of the various models.

1. Introduction

A common finding in much of the empirical finance literature is that although the returns on speculative assets contain little serial correlation, the absolute returns and their power transformations are highly correlated. In particular, Ding et al. (1993) investigate the autocorrelation structure of $|r_t|^{\delta}$, where r_t is the daily S&P 500 stock market return, and δ is a positive number. They find that $|r_t|$ has significant positive autocorrelations for long lags. Motivated by this empirical result they propose a new general class of ARCH models, which they call the Asymmetric Power ARCH (APARCH). In addition, they show that this formulation comprises seven other specifications in the literature.[2] For an in depth discussion of the theoretical properties of the APARCH model see Karanasos and Kim (2006). McCurdy and Michaud (1996) and Tse (1998) extend the asymmetric power formulation of the variance to incorporate fractional integration, as defined by Baillie et al. (1996).[3] The new specification is termed fractionally integrated APARCH (FIAPARCH).

☆ The final version of this paper was written while Ning Zeng was a PhD student at Brunel University. An earlier version of this paper by the first two authors circulated under the title "Fractionally Integrated APARCH Modeling of Stock Market Volatility: A multi-country study".

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2 These models are: the ARCH (Engle, 1982), the GARCH (Bollerslev, 1986), the Taylor/Schwert GARCH in standard deviation (Taylor, 1986, and Schwert, 1990), the GJR GARCH (Glosten et al., 1993), the TARCH (Zakoian, 1994), the NARCH (Higgins and Bera, 1992) and the log-ARCH (Geweke, 1986, and Pantula, 1986).

3 The FIGARCH model of Baillie et al. (1996) is closely related to the long-memory GARCH process introduced in Karanasos et al. (2003) (see also Conrad and Karanasos, 2006, and Conrad, forthcoming, and the references therein).

0927-5398/$ – see front matter
doi:10.1016/j.jempfin.2010.05.001

The FIAPARCH model increases the flexibility of the conditional variance specification by allowing (a) an asymmetric response of volatility to positive and negative shocks, (b) the data to determine the power of returns for which the predictable structure in the volatility pattern is the strongest, and (c) long-range volatility dependence. These three features in the volatility processes of asset returns have major implications for many paradigms in modern financial economics. For example, the pricing of long-term options and optimal portfolio allocations must take into account all of these three properties.

At the same time, the FIAPARCH specification possesses the useful property that it nests the formulation without power effects and the stable one as special cases. This provides an encompassing framework for these two broad classes of specifications and facilitates comparison between them. The main contribution of this paper is to enhance our understanding of whether and to what extent this type of model improves upon its simpler counterparts.

Brooks et al. (2000) provide evidence for the applicability of the univariate APARCH model to national stock market returns for ten countries plus a world index. The results by Tse (1998) suggest that the FIAPARCH model is applicable to the yen–dollar exchange rate. More recently, Degiannakis (2004) and Ñíguez (2007) have applied univariate FIAPARCH specifications to stock return data. So far, multivariate versions of the framework have rarely been used in the literature. Only Dark (2004) applies a bivariate error correction FIAPARCH model to examine the relationship between stock and future markets, and Kim et al. (2005) use a bivariate FIAPARCH-in-mean process to model the volume–volatility relationship. Therefore, an interesting research issue is to explore how generally applicable this formulation is to a wide range of financial data and whether multivariate specifications can outperform their univariate counterparts. In this paper we address this issue by estimating both univariate and multivariate versions of this framework for eight series of national stock market index returns. These countries are Canada, France, Germany, Hong Kong, Japan, Singapore, the United Kingdom and the United States. As the general multivariate specification adopted in this paper nests the various univariate formulations, the relative ranking of each of these models can be considered using the Wald testing procedures and standard information criteria. Furthermore, the ability of the FIAPARCH formulation to forecast stock volatility out-of-sample is assessed by a variety of forecast error statistics. In order to verify whether the difference between the statistics from the various models is statistically significant we employ the tests of Diebold and Mariano (1995) and Harvey et al. (1997).

The remainder of this article is structured as follows. In Section 2 we detail the univariate and multivariate FIAPARCH models and discuss the various nested ARCH specifications. Section 3 discusses the data and presents the empirical results. In Section 4 we evaluate the different specifications in terms of their out-of-sample forecast ability. Finally, Section 5 concludes the analysis.

2. FIAPARCH model

2.1. Univariate process

One of the most common models in finance and economics to describe a time series r_t of stock returns is the AR(1) process

$$(1-\zeta L)r_t = c + \varepsilon_t, \quad t \in \mathbb{N}, \tag{2.1}$$

with

$$\varepsilon_t = e_t\sqrt{h_t},$$

where $|c| \in [0, \infty)$, $|\zeta| < 1$ and $\{e_t\}$ are independently and identically distributed (*i.i.d.*) random variables with $E(e_t) = E(e_t^2 - 1) = 0$. h_t is positive with probability one and is a measurable function of Σ_{t-1}, which in turn is the sigma-algebra generated by $\{r_{t-1}, r_{t-2}, \dots\}$. That is h_t denotes the conditional variance of the returns $\{r_t\}$, i.e. $E[r_t|\Sigma_{t-1}] = c + \zeta r_{t-1}$ and $\mathrm{Var}[r_t|\Sigma_{t-1}] = h_t$.

Tse (1998) examines the conditional heteroskedasticity of the yen–dollar exchange rate by employing the FIAPARCH (1, d, 1) model. Accordingly, we utilize the following process

$$(1-\beta L)\left(h_t^{\delta/2} - \omega\right) = \left[(1-\beta L) - (1-\phi L)(1-L)^d\right](1 + \gamma s_t)|\varepsilon_t|^{\delta}, \tag{2.2}$$

where $\omega \in (0, \infty)$, $|\beta| < 1$, $|\phi| < 1$, $0 \le d \le 1$, $s_t = 1$ if $\varepsilon_t < 0$ and 0 otherwise, γ is the leverage coefficient, and δ is the parameter for the power term that takes (finite) positive values. A sufficient condition for the conditional variance h_t to be positive almost surely for all t is that $\gamma > -1$ and the parameter combination (ϕ, d, β) satisfies the inequality constraints provided in Conrad and Haag (2006) and Conrad (forthcoming).

When $d = 0$, the process in Eq. (2.2) reduces to the APARCH(1,1) one, which nests two major classes of ARCH models. Specifically, a Taylor/Schwert type of formulation is specified when $\delta = 1$, and a Bollerslev type is specified when $\delta = 2$. There seems to be no obvious reason why one should assume that the conditional standard deviation is a linear function of lagged absolute returns or the conditional variance a linear function of lagged squared returns. As Brooks et al. (2000, p. 378) point out "the common use of a squared term in this role ($\delta = 2$) is most likely to be a reflection of the normality assumption traditionally invoked regarding financial data. However, if we accept that (high frequency) data are very likely to have a non-normal error