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Optimal control of nonlinear dynamic econometric models: An algorithm and an application[☆]

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ABSTRACT

OPTCON is an algorithm for the optimal control of nonlinear stochastic systems which is particularly applicable to econometric models. It delivers approximate numerical solutions to optimum control problems with a quadratic objective function for nonlinear econometric models with additive and multiplicative (parameter) uncertainties. The algorithm was programmed in *C#* and allows for deterministic and stochastic control, the latter with open-loop and passive learning (open-loop feedback) information patterns. The applicability of the algorithm is demonstrated by experiments with a small quarterly macroeconometric model for Slovenia. This illustrates the convergence and the practical usefulness of the algorithm and (in most cases) the superiority of open-loop feedback over open-loop controls.

1. Introduction

Optimum control theory has a great number of applications in many areas of science from engineering to economics. In particular, there are many studies on determining optimal economic policies for econometric models. Most of these optimum control applications use algorithms for linear dynamic systems or those that do not take the full stochastic nature of the econometric model into account. Examples of the former are Kendrick (1981) and Coomes (1987), and the references in Amman (1996) and Chow (1975, 1981) for the latter. An algorithm that is explicitly aimed at providing (approximate) solutions to optimum control problems for nonlinear econometric models and other dynamic systems with different kinds of stochastics is OPTCON, as introduced by Matulka and Neck (1992). However, so far OPTCON has been severely limited by being based on very restrictive assumptions about the information available to the decision-maker. In particular, learning about the econometric model while in the process of controlling the economy was ruled out by assumption. In reality, however, new information arrives in each period, and econometric models are regularly re-estimated using this information. Therefore, extensions of the OPTCON algorithm to include various possibilities of obtaining and using new information about the system to be controlled are highly desirable.

The present extension of the OPTCON algorithm from open-loop control only (OPTCON1) to the inclusion of passive learning or open-loop feedback control where the estimates of the parameters are updated in each period results in the algorithm OPTCON2. It can deliver approximately optimal solutions to dynamic optimization (optimum control) problems for a rather large class of nonlinear dynamic systems under a quadratic objective function with stochastic uncertainty in the parameters and in the system equations under both kinds of control schemes. In the open-loop feedback part, it is assumed that new realizations of both random processes occur in each period, which can be used to update the parameter estimates of the dynamic system, i.e. of the econometric model. Following Kendrick's (1981) approach, the parameter estimates are

☆ The source code of the algorithm OPTCON is available from the authors on request.

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updated using the Kalman filter in order to arrive at more reliable approximations to the solution of stochastic optimum control problems. Whether this hope will materialize depends upon the comparative performance of open-loop feedback vs. open-loop control schemes in actual applications. Some indication of this will be provided by comparing the two control schemes within a control problem for a small econometric model. This also serves to show that the OPTCON2 algorithm and its implementation in C# actually deliver plausible numerical solutions, at least for a small problem, with real economic data.

The paper has the following structure. In Section 2, the class of problems to be tackled by the algorithm is defined. Section 3 briefly reviews the OPTCON1 algorithm. Section 4 explains the OPTCON2 algorithm. In Section 5, the small econometric model for Slovenia SLOVNL is introduced, the applicability and convergence of OPTCON2 as implemented in C# is shown, and the quality of open-loop and open-loop feedback (passive learning) controls in Monte Carlo simulations for this model are compared. Section 6 concludes. More details about the mathematics of the algorithm are given in Blueschke-Nikolaeva et al. (2010).

2. The problem

The OPTCON algorithm is designed to provide approximate solutions to optimum control problems with a quadratic objective function (a loss function to be minimized) and a nonlinear multivariate discrete-time dynamic system under additive and parameter uncertainties. The intertemporal objective function is formulated in a quadratic tracking form, which is quite often used in applications of optimum control theory to econometric models. It can be written as

$$J = E\left[\sum_{t=1}^{T} L_t(x_t, u_t)\right], \tag{1}$$

with

$$L_t(x_t, u_t) = \frac{1}{2}\begin{pmatrix} x_t - \tilde{x}_t \\ u_t - \tilde{u}_t \end{pmatrix}' W_t \begin{pmatrix} x_t - \tilde{x}_t \\ u_t - \tilde{u}_t \end{pmatrix}. \tag{2}$$

x_t is an n-dimensional vector of state variables that describes the state of the economic system at any point in time t. u_t is an m-dimensional vector of control variables, $\tilde{x}_t \in R^n$ and $\tilde{u}_t \in R^m$ are given 'ideal' (desired, target) levels of the state and control variables respectively. T denotes the terminal time period of the finite planning horizon. W_t is an $((n+m) \times (n+m))$ matrix, specifying the relative weights of the state and control variables in the objective function. In a frequent special case, W_t is a matrix including a discount factor α with $W_t = \alpha^{t-1} W$. W_t (or W) is symmetric.

The dynamic system of nonlinear difference equations has the form

$$x_t = f(x_{t-1}, x_t, u_t, \theta, z_t) + \varepsilon_t, \; t = 1, \ldots, T, \tag{3}$$

where θ is a p-dimensional vector of parameters whose values are assumed to be constant but unknown to the decision-maker (parameter uncertainty), z_t denotes an l-dimensional vector of non-controlled exogenous variables, and ε_t is an n-dimensional vector of additive disturbances (system error). θ and ε_t are assumed to be independent random vectors with expectations $\hat{\theta}$ and O_n respectively and covariance matrices $\Sigma^{\theta\theta}$ and $\Sigma^{\varepsilon\varepsilon}$ respectively. f is a vector-valued function, $f^i(.....)$ is the ith component of $f(.....)$, $i = 1, \ldots, n$.

3. OPTCON1

The basic OPTCON algorithm determines approximate solutions to optimum control problems with a quadratic objective function and a nonlinear multivariate dynamic system under additive and parameter uncertainties. It combines elements of previous algorithms developed by Chow (1975, 1981), which incorporate nonlinear systems but no multiplicative uncertainty, and Kendrick (1981), which deals with linear systems and all kinds of uncertainty. The version OPTCON1 is described in detail in Matulka and Neck (1992); here only its basic idea is presented.

It is well known in stochastic control theory that a general analytical solution to dynamic stochastic optimization problems cannot be achieved even for very simple control problems. The main reason is the so-called dual effect of control under uncertainty, meaning that controls not only contribute directly to achieving the stated objective but also affect future uncertainty and hence the possibility of indirectly improving on the system performance by providing better information about the system (see, for instance, Aoki, 1989; Neck, 1984). Therefore only approximations to the true optimum for such problems are feasible, with various schemes existing to deal with the problem of information acquisition.

A useful distinction was adapted from the control engineering literature by Kendrick (1981): open-loop policies preclude the possibility of receiving information (measurements) while the system is in operation; open-loop feedback (or passive learning) policies use current information to determine the control but do not anticipate future measurements; and closed-loop (or active learning) policies make some use of information about future measurements as well. Alternatively, Kendrick and Amman (2006) propose the terms optimal feedback and expected optimal feedback for open-loop and open-loop feedback respectively. Given the intricacies of the interplay between control and information, even for very simple stochastic