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# Market structure and matching with contracts ☆

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**Abstract**

Ostrovsky (2008) [9] develops a theory of stability for a model of matching in exogenously given networks. For this model a generalization of pairwise stability, *chain stability*, can always be satisfied as long as agents' preferences satisfy same side substitutability and cross side complementarity. Given this preference domain I analyze the interplay between properties of the network structure and (cooperative) solution concepts. The main structural condition is an acyclicity notion that rules out the implementation of trading cycles. It is shown that this condition and the restriction that no pair of agents can sign more than one contract with each other are jointly necessary and sufficient for (i) the equivalence of group and chain stability, (ii) the core stability of chain stable networks, (iii) the efficiency of chain stable networks, (iv) the existence of a group stable network, and (v) the existence of an efficient and individually stable network. These equivalences also provide a rationale for chain stability in the unrestricted model. The (more restrictive) conditions under which chain stability coincides with the core are also characterized.


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## 1. Introduction

Theoretical models of network formation and matching markets are concerned with predicting which outcomes are likely to emerge when self-interested agents interact. An important strand of this literature belongs to the area of cooperative game theory and "likely outcomes" are not defined by writing down an explicit negotiation protocol, but rather by postulating a set of stability constraints that one perceives to be relevant in the problem under study. In several cases such constraints have been an important guideline for the design of real-life mechanisms for *two-sided matching problems* in which a group of workers/students has to be assigned among a set of firms/schools (see [12] and the references therein). The literature has focused on the *pairwise stability* concept, which only considers the possibility of coordinated deviations by pairs of players. As long as workers can take at most one job and firms have substitutable preferences, a pairwise stable matching not only exists [6], but is also *group stable* [13]: There is no group of agents who can obtain a strictly preferred matching by forming new partnerships only among themselves, possibly dropping some previously held partnerships. In particular, a pairwise stable matching is *efficient*. While these are encouraging results for a restricted class of assignment problems, many interesting applications do not fit the assumptions above: Some workers may demand multiple jobs in a labor market,[1] firms may not view workers as substitutes,[2] and markets are often not two-sided.[3]

Ostrovsky [9] shows how some of these features can be accommodated by a model in which agents are located in an exogenously given vertically ordered directed network and have preferences over sets of trading relationships, or *contracts*, with their neighbors. A set of contracts is *chain stable* if (i) it is *individually stable* in the sense that no agent would prefer to drop some of her contracts, and (ii) there is no downstream sequence of agents who can obtain a strictly preferred set of contracts by forming new contracts only with their direct neighbors in the sequence, possibly dropping some of their previously held contracts. Ostrovsky shows that chain stable outcomes exist as long as agents' preferences satisfy *same side substitutability* and *cross side complementarity*. However, unlike pairwise stable matchings in the simple matching models above, chain stable allocations may not be group stable. In fact, chain stable outcomes may even be inefficient and thus fail to be in the *core*.[4] I characterize the conditions under which these problems cannot occur.

A main contribution of this study is methodological. Instead of introducing further restrictions on preferences, I introduce restrictions on *who can contract with whom* and *how many relationships agents can form*. Such constraints are common in two-sided matching models, where an agent cannot contract with another agent on her side of the market and it is often assumed that agents on one side of the market can all engage in at most one relationship. The restrictions that I develop can be interpreted as restricting the sets of acceptable contracts. However, unlike the usual preference restrictions in the matching literature, such as responsiveness or strong

[1] For example, Echenique and Oviedo [3] mention that 35% of teachers in Argentina work for more than one school.

[2] Many tasks can only be accomplished by the combined workforce of a set of specialized workers. The construction of a building, for example, requires a structural engineer, a carpenter, and so on, so that complementarities between individual workers are likely.

[3] Brokers act as intermediaries between owners and potential tenants in housing markets, temporary employment agencies supply firms with short-term labor, some stores (e.g. Gamestop) allow customers to trade in used goods which they then sell to other customers, and so on.

[4] An outcome is in the *core* (defined by weak domination), if no group of agents can obtain a weakly preferred outcome for all involved by trading only among themselves.