Physica A 391 (2012) 4883–4890

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Permit allocation in emissions trading using the Boltzmann distribution

Ji-Won Park [a,*], Chae Un Kim [b], Walter Isard [a,c,1]

[a] Regional Science, Cornell University, Ithaca, NY 14853, USA
[b] Cornell High Energy Synchrotron Source (CHESS), Cornell University, Ithaca, NY 14853, USA
[c] Economics, Cornell University, Ithaca, NY 14853, USA

ARTICLE INFO

Article history:
Received 1 December 2011
Received in revised form 24 February 2012
Available online 27 May 2012

Keywords:
Climate change
Emissions trading
Permit allocation
Entropy maximization
Fair distribution

ABSTRACT

In emissions trading, the initial allocation of permits is an intractable issue because it needs to be essentially fair to the participating countries. There are many ways to distribute a given total amount of emissions permits among countries, but the existing distribution methods, such as auctioning and grandfathering, have been debated. In this paper we describe a new method for allocating permits in emissions trading using the Boltzmann distribution. We introduce the Boltzmann distribution to permit allocation by combining it with concepts in emissions trading. We then demonstrate through empirical data analysis how emissions permits can be allocated in practice among participating countries. The new allocation method using the Boltzmann distribution describes the most probable, natural, and unbiased distribution of emissions permits among multiple countries. Simple and versatile, this new method holds potential for many economic and environmental applications.

1. Introduction

Scientists have warned that global warming of more than 1 °C would constitute a dangerous climate change based on the likely effects on sea levels and the extermination of species [1]. Furthermore, climate change that occurs as a result of increases in CO_2 concentration is largely irreversible for 1000 years, even after CO_2 emissions cease [2]. According to the *Stern Review,* prompt, decisive action is clearly warranted, and, because climate change is a global problem, the response to it must be international [3].

Various ideas have been proposed for slowing global warming and reducing CO_2 emissions into the atmosphere, including reflecting solar radiation with small particles in the stratosphere, putting deflectors in space, growing trees and other biomass to remove CO_2 from the atmosphere, fertilizing oceans with iron to remove CO_2, and reducing CO_2 emissions through carbon taxes or emissions trading [4]. Among these, numerous studies found that emissions trading lowers the cost of reaching the commitments of the Kyoto Protocol [5].

The basic concepts of emissions trading were established in the past decades [6–13]. As with any trading system, in the emissions trading system, the flow and value of what is traded depends on its initial allocation, its supply, and the demand for it [14]. There are many possible ways to distribute a given total of emissions permits among participants [15]; traditionally, grandfathering and auctioning have been suggested for initial permit allocation [16,17].

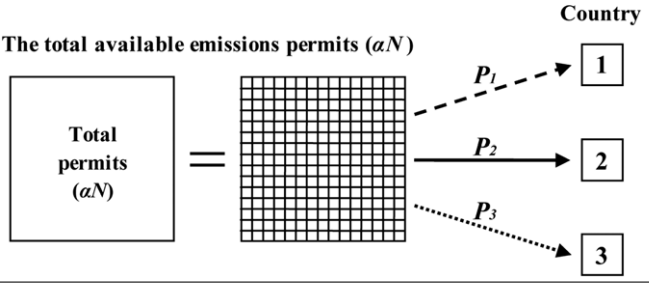
Permit allocation is one of the most intractable issues to resolve in designing emissions trading systems. A permit-allocation rule should be simple, should be based in part on historical data, and should be perceived as fair [18]. Because the

* Corresponding author.
E-mail address: jp429@cornell.edu (J.-W. Park).

[1] Professor Isard passed away before the submission of this manuscript at the age of 91.

0378-4371/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2012.05.052

Table 1
The Boltzmann distribution for permit allocation.

Boltzmann distribution	Description
In physical sciences	$P_i \propto e^{-\beta E_i}$ Where, P_i = probability that a particle stays in substate i e = constant of the exponential function ≈2.71828 $\beta = 1/kT$ (k = Boltzmann constant, T = absolute temperature) E_i = energy of substate i
Potential application in permit allocation	$P_i \propto C_i e^{-\beta E_i}$ Where, P_i = probability that emissions permits are allocated to a country i e = constant of the exponential function ≈2.71828 β = constant (≥ 0) E_i = allocation potential energy per capita of a country i C_i = total population of a country i
Permit allocation using the Boltzmann distribution*	

* The total available emissions permits (αN) are split into N pieces of unit carbon credit α, and then the emissions permits are allocated to country i ($i = 1, 2,$ and 3) based on the probability distribution (P_i) from the Boltzmann distribution. Note that the number (N) of unit emissions permits can always be made large enough for the Boltzmann statistics by making the unit emissions permit (α) smaller.

flow and value of emissions permits depends on their initial allocation, the fair allocation of a limited number of permits among countries or firms is not only important but also controversial.

In this paper, we introduce an alternative method for initial permit allocation using the Boltzmann distribution. We first describe the basic concept of the Boltzmann distribution and then develop its mathematical formula for the allocation of emissions permits. Next, through empirical data analysis, we demonstrate how this allocation method can be used in practice for initial permit allocation.

2. The Boltzmann distribution

In the physical sciences, the Boltzmann distribution yields the equilibrium probability distribution of a physical system in its energy substates [19,20]. The description is valid as long as each physical particle of the system is identical to but distinguishable from the others and as long as the interaction among the particles can be taken to be negligible. Based on the Boltzmann distribution, the probability (P_i) that a particle can be found in the ith substate is inversely proportional to the exponential function of the substate energy (E_i). A well-known example is the Maxwell–Boltzmann distribution, which describes the velocity distribution of ideal gas molecules [19–21].

The Boltzmann distribution is based on entropy maximization and has been employed in a number of fields. For example, some economists and physicists have introduced entropy concepts into the field of economics and have discussed the distribution of economic systems and their evolution [22–34]. Similarly, econophysicists have employed a stochastic process in describing the dynamics of individual wealth or income and in deriving their probability distributions [33,35–38].

In this paper, entropy maximization is brought to international emissions trading via the Boltzmann distribution, which provides guidelines for allocating emissions permits among multiple countries. Here, the concept of a physical system is replaced by the concept of an emissions trading system that consists of all participating countries. The concept of the physical particle is replaced by that of the unit emissions permit. The concept of the physical substates is replaced by that of individuals of the participating countries. Assuming that all individuals in a country i contribute equally to the total CO_2 emissions of that country i, the potential energy E_i of a physical substate i is replaced by the "allocation potential energy per capita" (E_i) of the country i. With this replacement, the E_i becomes an *intensive* variable (as apposed to an *extensive* variable) for the country i [19]. Thus the probability that a unit emissions permit is allocated to a country i is proportional to its total population and is inversely proportional to the exponential function of the allocation potential energy per capita E_i. This allocation potential energy per capita may be given in complicated forms and can include various political and economic parameters. One simplest form can be given in a way such that it is negatively proportional to the actual CO_2 emissions per capita of a country i. Details on the probability distribution function are summarized in Table 1. Table 1 also includes a diagram of the permit allocation using the Boltzmann distribution.