ARTICLE IN PRESS
YJETH:4164

Available online at www.sciencedirect.com

SciVerse ScienceDirect

Journal of Economic Theory ••• (••••) •••–•••

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Equilibria of dynamic games with many players: Existence, approximation, and market structure

Sachin Adlakha [a,*], Ramesh Johari [b], Gabriel Y. Weintraub [c]

[a] *Center for the Mathematics of Information, California Institute of Technology, Pasadena, CA 91125, Unites States*
[b] *Department of Management Science and Engineering, Stanford University, Stanford, CA 94305, United States*
[c] *Columbia Business School, Columbia University, New York, NY 10027, United States*

Received 2 December 2011; final version received 29 January 2013; accepted 27 June 2013

Abstract

In this paper we study stochastic dynamic games with many players; these are a fundamental model for a wide range of economic applications. The standard solution concept for such games is *Markov perfect equilibrium* (MPE), but it is well known that MPE computation becomes intractable as the number of players increases. We instead consider the notion of *stationary equilibrium* (SE), where players optimize assuming the empirical distribution of others' states remains constant at its long run average. We make two main contributions. First, we provide a rigorous justification for using SE. In particular, we provide a parsimonious collection of exogenous conditions over model primitives that guarantee existence of SE, and ensure that an appropriate approximation property to MPE holds, in a general model with possibly unbounded state spaces. Second, we draw a significant connection between the validity of SE, and market structure: under the same conditions that imply SE exist and approximates MPE well, the market becomes fragmented in the limit of many firms. To illustrate this connection, we study in detail a series of dynamic oligopoly examples. These examples show that our conditions enforce a form of "decreasing returns to larger states;" this yields fragmented industries in the SE limit. By contrast, violation of these conditions suggests "increasing returns to larger states" and potential market concentration. In that sense, our work uses a fully dynamic framework to also contribute to a longstanding issue in industrial organization: understanding the determinants of market structure in different industries.

JEL classification: C62; C73; L11; L13

* Corresponding author.
E-mail addresses: adlakha@caltech.edu (S. Adlakha), ramesh.johari@stanford.edu (R. Johari), gweintraub@columbia.edu (G.Y. Weintraub).

0022-0531/$ – see front matter
http://dx.doi.org/10.1016/j.jet.2013.07.002

Please cite this article in press as: S. Adlakha et al., Equilibria of dynamic games with many players: Existence, approximation, and market structure, J. Econ. Theory (2013), http://dx.doi.org/10.1016/j.jet.2013.07.002

Keywords: Stochastic dynamic games; Approximation; Stationary equilibrium; Industrial organization

1. Introduction

A common framework to study dynamic economic systems of interacting agents is a *stochastic game*, as pioneered by Shapley [40]. In a stochastic game agents' actions directly affect underlying state variables that influence their payoff. The state variables evolve according to a Markov process in discrete time, and players maximize their infinite horizon expected discounted payoff. Stochastic games provide a valuable general framework for a range of economic settings, including *dynamic oligopolies*—i.e., models of competition among firms over time. In particular, since the introduction of the dynamic oligopoly model of Ericson and Pakes [17], they have been extensively used to study industry dynamics with heterogeneous firms in different applied settings (see [14] for a survey of this literature).

The standard solution concept for stochastic games is *Markov perfect equilibrium* (MPE) [21], where a player's equilibrium strategy depends on the current state of all players. MPE presents two significant obstacles as an analytical tool, particularly as the number of players grows large. First is *computability*: the state space expands in dimension with the number of players, and thus the "curse of dimensionality" kicks in, making computation of MPE infeasible in many problems of practical interest. Second is *plausibility*: as the number of players grows large, it becomes increasingly difficult to believe that individual players track the exact behavior of the other agents.

To overcome these difficulties, previous research has considered an asymptotic regime in which the number of agents is infinite [30,27]. In this case, individuals take a simpler view of the world: they postulate that fluctuations in the empirical distribution of other players' states have "averaged out" due to a law of large numbers, and thus they optimize holding the state distribution of other players fixed. Based on this insight, this approach considers an equilibrium concept where agents optimize only with respect to the long run average of the distribution of other players' states; Hopenhayn [27] refers to this concept as *stationary equilibrium* (SE), and we adopt his terminology. SE are much simpler to compute and analyze than MPE, making this a useful approach across a wide range of applications. In particular, SE of *infinite models* have also been extensively used to study industry dynamics (see, for example, [33,35,31,26]).

In this paper, we address two significant questions. First, *under what conditions is it justifiable to use SE as a modeling tool*? We provide theoretical foundations for the use of SE. In particular, our main results provide a parsimonious collection of *exogenous conditions over model primitives* that guarantee existence of SE, and ensure that an appropriate approximation property holds. These results provide a rigorous justification for using SE of infinite models to study stochastic games with a large but finite number of players.

The second question we address relates to a longstanding topic of research in industrial organization: *when do industries fragment, and when do they concentrate?* In a fragmented industry all firms have small market shares, with no single firm or group of firms becoming dominant. By contrast, in a concentrated industry, a few participants that hold a notable market share can exert significant market power. In dynamic oligopoly models in particular, this is a challenging question to answer due to the inherent complexity of MPE. Our second main contribution is to draw a significant connection between the validity of SE, and market structure: under the same conditions that imply SE exist and an appropriate approximation property holds, under all SE the market becomes fragmented in the limit of many firms. In particular, we interpret our con-