Insurance: Mathematics and Economics 46 (2010) 493–499

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

A risk tolerance model for portfolio adjusting problem with transaction costs based on possibilistic moments

Wei-Guo Zhang*, Xi-Li Zhang, Wei-Jun Xu

School of Business Administration, South China University of Technology, Guangzhou, 510641, PR China

ARTICLE INFO

Article history:
Received March 2009
Received in revised form
November 2009
Accepted 30 January 2010

Keywords:
Portfolio adjusting
Possibilistic moment
Transaction costs
SMO-type decomposition method

ABSTRACT

Due to changes of situation in financial markets and investors' preferences towards risk, an existing portfolio may not be efficient after a period of time. In this paper, we propose a possibilistic risk tolerance model for the portfolio adjusting problem based on possibility moments theory. A Sequential Minimal Optimization (SMO)-type decomposition method is developed for finding exact optimal portfolio policy without extra matrix storage. We present a simple method to estimate the possibility distributions for the returns of assets. A numerical example is provided to illustrate the effectiveness of the proposed models and approaches.

1. Introduction

In conventional portfolio analysis, a financial asset is usually characterized as a random variable with a probability distribution over its returns (Markowitz, 1959, 1987). The key principle of the mean–variance methodology for portfolio selection is that the investment return is measured by the mean and the investment risk is measured by the variance (or standard deviation). The mean–variance models are usually described mathematically in two ways: maximizing return when a level of risk is given and minimizing risk when a level of return is given. To formulate the models of portfolio selection, it is necessary to estimate the probability distribution, strictly speaking, a mean vector and a covariance matrix. It means that all expected returns, variances, and covariances of risky assets can be accurately estimated by an investor. However, the returns of risky assets are in an uncertain economic environment and vary from time to time. The future states of returns and risks of risky assets cannot be predicted accurately. So it is impossible for investors to get the precise probability distribution. Based on the idea of approximations, the mean–variance model of portfolio selection is extended (Liu, 2004; Samuelson, 1970).

Though probability theory is one of the main tools used for analyzing uncertainty in finance, it cannot describe uncertainty completely since there are some other uncertain factors that differ from the random ones found in financial markets. Non-probabilistic factors affect the financial markets such that the return of risky asset is fuzzy uncertain. Recently, a number of researchers investigated the fuzzy portfolio selection problem. Watada (1997) and León et al. (2002) presented two approaches for portfolio selection using fuzzy decision theory. Inuiguchi and Tanino (2000) introduced a possibilistic programming approach to the portfolio selection problem based on the minimax regret criterion. Lai et al. (2002) and Giove and Funari (2006) constructed interval programming models of portfolio selection. Zhang and Nie (2004), Zhang et al. (2006) and Zhang and Wang (2008) studied the admissible efficient portfolio selection problems under the assumption that the expected return and risk of asset have admissible errors to reflect the uncertainty in real investment actions, the admissible portfolio selection models are extensions of the conventional mean–variance models. Liu et al. (2006) proposed a linear belief function (LBF) approach to evaluate portfolio performance. Carlsson et al. (2002) introduced a possibilistic approach to selecting portfolios with highest utility score under the assumption that the returns of assets are trapezoidal fuzzy numbers. On the basis of the (crisp) possibilistic mean and possibilistic variance introduced by Carlsson and Fullér (2001), Zhang et al. (2009) dealt with the portfolio selection problem when the returns of assets obey LR-type possibility distributions and there exist limits on holdings. Zhang and Xiao (2009) proposed the weighted lower and upper possibilistic portfolio selection models with tolerated risk level, where the return rates of risky assets are trapezoidal fuzzy numbers. Zhang et al. (2007) proposed an algorithm which can derive the explicit expression of the possibilistic efficient frontier based on the return rates of risky assets with general possibility distributions. Based on Carlsson et al. (2002), Zhang et al. (2009) discussed the portfolio selection problem for bounded assets with the maximum possibilistic mean–variance utility. Li and Xu (2007) proposed a new portfolio selection model in a hybrid uncertain environment under the assumption that the returns of securities are fuzzy random variables.

* Corresponding author. Tel.: +86 20 87114121; fax: +86 20 87114105.
E-mail addresses: wgzhang@scut.edu.cn, zhwg61@263.net (W.-G. Zhang).

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doi:10.1016/j.insmatheco.2010.01.007

The above-mentioned portfolio selection models only considered how to select the optimal portfolio at the beginning of investment period. Due to changes of situation in financial markets and the investors' preferences towards risk, an existing portfolio may not be efficient after a period of time. Thus, the investors would consider to adjust the existing portfolio by buying or selling assets in response to changing conditions in financial markets. Therefore, the portfolio adjusting problem for an existing portfolio is an important issue for researchers and investors. Furthermore, this adjusting will incur a certain amount of transaction cost. Arnott and Wanger (1990) and Yoshimoto (1996) found that ignoring transaction costs would result in an inefficient portfolio. The purpose of this paper is to discuss the portfolio adjusting problem for an existing portfolio under the assumptions that the uncertain returns of assets in financial markets are fuzzy numbers. The contribution is in the following two aspects. This paper presents a risk tolerance model with transaction costs for adjusting an existing portfolio, which uses possibilistic moments of fuzzy number provided by Saeidifar and Pasha (2009). Meanwhile, this paper also proposes a Sequential Minimal Optimization(SMO)-type decomposition method for finding exact optimal portfolio policy without extra matrix storage.

The rest of this paper is organized as follows. We recall the notions of possibilistic moments of fuzzy numbers in Section 2. We propose the possibilistic risk tolerance model for the portfolio adjusting problem based on possibilistic moments and transform the problem into several special cases in Section 3. We design a SMO-type decomposition method specially for the portfolio adjusting problem in Section 4. We present a simple method to estimate the possibility distributions and give a numerical example to illustrate the proposed model and algorithm in Section 5. Finally, some concluding remarks are given in Section 6.

2. Preliminaries

Let us introduce some definitions which are needed in the following section. A fuzzy number A is a fuzzy set of the real line $\mathcal{R}$ with a normal, fuzzy convex and continuous membership function of bounded support. The family of fuzzy numbers is denoted by $\mathcal{F}$. A fuzzy number A with γ-level set is expressed as $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$ $(\gamma > 0)$.

The r-th possibilistic moment about zero of a fuzzy number is introduced as (Saeidifar and Pasha, 2009):

$$\mu'_r(A) = \int_0^1 \gamma[(a_1(\gamma))^r + (a_2(\gamma))^r]\mathrm{d}\gamma, \quad r = 1, 2, \ldots,$$

where the first possibilistic moment $\mu'_1(A) = \int_0^1 \gamma[a_1(\gamma) + a_2(\gamma)]\mathrm{d}\gamma$ is defined as the crisp possibilistic mean value of A (Carlsson and Fullér, 2001; Saeidifar and Pasha, 2009). Saeidifar and Pasha (2009) showed that $\mu'_1(A)$ is the nearest weighted point to $A \in \mathcal{F}$ which is unique. Furthermore it is a new and interesting alternative justification to the definition of the weighted mean value of a fuzzy number that is defined in Carlsson and Fullér (2001).

The r-th possibilistic moment about the possibilistic mean value of A is introduced as (Saeidifar and Pasha, 2009):

$$\mu_r(A) = \int_0^1 \gamma\left([\mu'_1(A) - a_1(\gamma)]^r + [\mu'_1(A) - a_2(\gamma)]^r\right)\mathrm{d}\gamma.$$

Correspondingly, the second possibilistic moment $\mu_2(A)$ is defined as the possibilistic variance of A (Carlsson and Fullér, 2001; Saeidifar and Pasha, 2009), where

$$\mu_2(A) = \int_0^1 \gamma\left([\mu'_1(A) - a_1(\gamma)]^2 + [\mu'_1(A) - a_2(\gamma)]^2\right)\mathrm{d}\gamma.$$

It is straightforward to show the following identity:

$$\mu_2(A) = \mu'_2(A) - (\mu'_1(A))^2.$$

The variance of fuzzy number is defined as the possibility-weighted average of the squared distance between the mean value and the left-hand and right-hand endpoints of its level sets. The variance is always positive and a measure of dispersion or spread of the fuzzy number. In the physical interpretation of the variance, it gives the moment of inertia of the mass distributed about the center of mass, also the variance gives information about the spread of variables around the mean value and it is a very important factor to find out the fluctuation in the observed values (more see Carlsson and Fullér, 2001; Saeidifar and Pasha, 2009).

A fuzzy number A is called trapezoidal with tolerance interval $[a, b]$, left-width $\alpha > 0$ and right-width $\beta > 0$ if its membership function has the following form:

$$A(u) = \begin{cases} 1 - \dfrac{a-u}{\alpha} & \text{if } a - \alpha \le u \le a, \\ 1 & \text{if } u \in [a, b], \\ 1 - \dfrac{u-b}{\beta} & \text{if } b \le u \le b + \beta, \\ 0 & \text{otherwise.} \end{cases}$$

The above trapezoidal fuzzy number A is usually denoted by the notation $A = (a, b, \alpha, \beta)$, and the γ-level set of A can be computed as

$$[A]^\gamma = [a - (1-\gamma)\alpha, b + (1-\gamma)\beta], \quad \forall \gamma \in [0, 1].$$

It is easy to see that the crisp possibilistic mean value and the possibilistic variance of A can be quantified as

$$\mu'_1(A) = \frac{a+b}{2} + \frac{\beta - \alpha}{6}$$

and

$$\mu_2(A) = \left[\frac{b-a}{2} + \frac{\alpha+\beta}{6}\right]^2 + \frac{(\alpha+\beta)^2}{72} + \frac{(\alpha-\beta)^2}{72},$$

respectively.

3. A possibilistic risk tolerance model for the portfolio adjusting problem

In this section, we discuss the portfolio adjusting problem using possibilistic moments. Suppose that an investor considers portfolio selection with n risky assets. In order to describe it conveniently, we use the following notations:

r_i, the return rate of risky asset i;
x_i, the proportion invested in risky asset i;
u_i, the upper bound constraint on x_i, $0 < u_i \le 1$;
$i = 1, \ldots, n$.

We assume the transaction cost is a **V**-shaped function of differences between a new portfolio $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and the existing portfolio $\mathbf{x}_0 = (x_1^0, x_2^0, \ldots, x_n^0)$. So the transaction cost of asset i can be expressed by

$$TC_i = c_i|x_i - x_i^0|,$$

where c_i is unit transaction cost rate for the risky asset i, $i = 1, \ldots, n$.

Thus, the total transaction cost can be written as

$$\sum_{i=1}^n TC_i = \sum_{i=1}^n c_i|x_i - x_i^0|, \quad i = 1, \ldots, n.$$

The net return r on the portfolio $(x_1, \ldots, x_n)$ after paying transaction costs is given by

$$r = \sum_{i=1}^n r_i x_i - \sum_{i=1}^n c_i|x_i - x_i^0|.$$

It is well known that the future states of returns and risks about risky assets are hard to predict accurately. In many important cases, the estimation of the possibility distributions of return rates on assets may be easier than the probability distributions.