Available online at www.sciencedirect.com
ScienceDirect
Computers & Industrial Engineering 52 (2007) 41–56
computers & industrial engineering
www.elsevier.com/locate/dsw

Job scheduling to minimize the weighted waiting time variance of jobs ☆

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Received 3 February 2006; received in revised form 13 August 2006; accepted 13 October 2006
Available online 22 December 2006

Abstract

This study considers the job scheduling problem of minimizing the weighted waiting time variance (WWTV) of jobs. It is an extension of WTV minimization problems in which we schedule a batch of n jobs, for servicing on a single resource, in such a way that the variance of their waiting times is minimized. WWTV minimization finds its applications for job scheduling in manufacturing systems with earliness and tardiness (E/T) penalties, in computer and networks systems for the stabilized QoS, and in other fields where it is desirable to minimize WWTV of jobs with different weights for priorities. We formulate a WWTV problem as an integer programming problem, prove the V-shape property for agreeably weighted WWTV problems and the nondelay property for general WWTV problems, and discover the strong V-Shape tendency of the optimal job sequences for this problem. Two job scheduling algorithms, Weighted Verified Spiral (WVS) and Weighted Simplified Spiral (WSS), are developed for the WWTV problems. Numerical testing shows that WVS and WSS significantly outperform existing WWTV algorithms.

Keywords: Weighted waiting time variance (WWTV); Quality of service (QoS); Just-in-time (JIT); Job scheduling

1. Introduction

The single machine weighted waiting time variance (WWTV) minimization problem, denoting by $1\|WWTV$, is to schedule the jobs in a batch on a single machine so as to minimize the weighted waiting time variance of the jobs as follows:

$$\min_{\lambda\in\Pi} \mathrm{WWTV} = \frac{1}{n-1}\sum_{i=1}^{n} v_i(\lambda)[W_i(\lambda) - \overline{W}(\lambda)]^2 \tag{1}$$

☆ This manuscript was processed by Area Editor Maged M. Dessouky.

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0360-8352/$ - see front matter
doi:10.1016/j.cie.2006.10.010

where Π is the set of all the permutations of the n jobs, $v_i(\lambda)$ is the weight of the job on position i to indicate its priority, $W_i(\lambda)$ is the waiting time of the job on position i and $\overline{W}(\lambda)$ is the weighted average waiting time of the jobs for a given sequence λ. A greater value of v_i means a higher priority. It is an extension of the WTV problem in which jobs are considered to have the same weight (Merten & Muller, 1972; Ye, Li, Xu, & Farley, 2006).

WWTV minimization has many applications in different fields. WWTV problems are closely related to weighted common due date problems. A weighted common due date problem can be modeled as finding an optimal common due date, d, to minimize the mean squared deviation (MSD), $\sum v_i(W_i - d)^2$. It can be obtained as $(\sum v_i^* W_i^*)/\sum v_i^*$, where v_i^* and W_i^* are the weight and waiting time of the jobs in the optimal WWTV sequence, respectively, since MSD is the second moment about d. The connection between the WWTV minimization problems and the weighted common due date problems suggests the applicability of putting WWTV minimization for scheduling problems with earliness and tardiness (E/T) penalties in manufacturing systems (Baker & Scudder, 1990; Cheng & Gupta, 1989; Verma & Dessouky, 1998). This is an extension of the relationship between the WTV problem and common due date problem or mean squared deviation (MSD) problem (Bagchi, Sullivan, & Chang, 1987; Cheng & Gupta, 1989; Cheng & Kovalyov, 1996; Cheng, Chen, & Shakhlevich, 2002; Panwalker, Smith, & Seidmann, 1982). It is shown that the CTV problem is equivalent to the unconstrained version of the mean squared deviation minimization problem (Bagchi et al., 1987).

In addition to applications to job scheduling for E/T minimization in manufacturing systems, WWTV is in connection with providing stabilized Quality of Service (QoS) in computers and networks (Ye et al., 2006). For instance, the data packets from WTV-sensitive applications (audio or video players) should have higher priorities of data transmission against other data packets (emailing or web browsing) arriving at a router. It is desirable to minimize the weighted WTV of the data packets to stabilize the router so that higher priority data packets get more consistent service performance or stable QoS. WWTV minimization may also find application in other service facilities where it is desirable to serve the jobs with different weights in stability.

The authors (Merten & Muller, 1972) first introduce the weighted variance minimization for file organization problems in which it is desirable to treat user's requests to data file in a stable manner. They propose a problem model that involves arbitrary job processing times and weights. One special case of the model, in which the jobs have equal weights, has received extensive studies since then. The authors (Merten & Muller, 1972) find that a sequence minimizing the variance of flow-time or completion time is antithetical to a sequence minimizing the variance of waiting time if the jobs have the same weight. An optimality condition of a WTV problem is shown in (Cai, 1996; Eilon & Chowdhury, 1977; Mittenthal, Raghavachari, & Rana, 1995) that the optimal sequence must be V-shaped, which means that the jobs before the job with the shortest processing time are sorted in a descending order while the jobs after the smallest job are sorted in an ascending order. The author (Schrage, 1975) considered the optimal sequences for problems with up to five jobs and showed that the longest job must be the first to be processed to minimize the completion time variance (CTV). The author (Schrage, 1975) conjectured the positions of the three longest jobs which were proven in (Hall & Kubiak, 1991). A counter example shows that the conjecture about the fourth longest job is incorrect in (Kanet, 1981). The author in (Kubiak, 1993) proves that the CTV problem on a single machine is NP-hard. The bounds for the position of the smallest job in the CTV problem are established in (Manna & Prasad, 1999). The author in (Kubiak, 1995) formulates the CTV problem as a problem of maximizing a zero-one quadratic function which is a sub-modular function with a special cost structure. The variance of job completion time with bi-criteria extension is investigated in (De, Ghosh, & Wells, 1992, 1996). A branch and bound algorithm to minimize CTV is given in (Viswanathkumar & Srinivasan, 2003) and a tabu search-based solution is developed in (Al-Turki, 2001). The pseudo polynomial algorithms and fast polynomial approximation schemes for CTV minimization problems are given in (Cheng & Kovalyov, 1996; Kubiak, Cheng, & Kovalyov, 2002; Manna & Prasad, 1997). A sufficient optimality condition for stochastic CTV is discussed in (Cai, 1996). More heuristic methods are developed for CTV/WTV problems in (Al-Turki, Fediki, & Andijani, 2001; Eilon & Chowdhury, 1977; Gowrishankar, Rajendran, & Srinivasan, 2001; Kanet, 1981; Sharma, 2002; Vani & Raghavachari, 1987; Ye et al., 2006).

We can see that there are extensive studies in the literature about WTV problems. However, studies on the WWTV problem are limited. The author in (Cai, 1995) considers the minimization of "agreeably weighted" completion time variance on a single machine. The "agreeably weighted" setting means that $p_i < p_j$ implies $v_i \geqslant v_j$, where p_i, v_i, p_j and v_j are the processing time and the weight of job i and j, respectively. This indicates