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Thermal criticality in viscous reactive flows through channels with a sliding wall: An exploitation of the Hermite–Padé approximation method

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Abstract

In this paper, we study the steady state solutions for viscous reactive flows through channels with a sliding wall. The reaction is assumed to be strongly exothermic under Arrhenius kinetics, neglecting the consumption of the material. Approximate solutions are constructed for the governing nonlinear boundary value problem using regular perturbation techniques together with a special type of Hermite–Padé approximants, and important properties of the temperature field including bifurcations and thermal criticality conditions are discussed.

Keywords: Lubricant hydrodynamics; Sliding wall; Hermite–Padé approximants

1. Introduction

The study of the thermal effect of a sliding plate on a viscous reacting fluid is extremely important in understanding lubricants hydrodynamics in engineering systems as well as plasma and fluid physics [1,2,5,8,12]. Lubricant is a thin viscous film used to prevent solid-to-solid contact during sliding motion. Generally speaking, most lubricants used in both engineering and industrial processes are reactive e.g. hydrocarbon oils, polyglycols, synthetic esters, polyphenylethers, etc., and their efficiency depends largely on the temperature variation from time to time. Hence, it is important to determine the thermal criticality conditions for viscous reactive fluid effectiveness as lubricants. In this particular problem, we have assumed that the pressure gradient is zero and the flow is driven solely by uniform velocity at the upper plate, i.e. the well-known plane Couette flow [2]. The resulting velocity profile is linear with zero value at the lower fixed plate and maximum value at the upper moving plate. Generally, when a fluid is sheared, some of the work done is dissipated as heat. The shear-induced heating gives an inevitable increase in temperature within the fluid. Neglecting the reacting viscous fluid consumption, since the channel is narrow (see Fig. 1) with very small aspect ratio, the equations for the momentum and heat balance in one dimension together with the boundary

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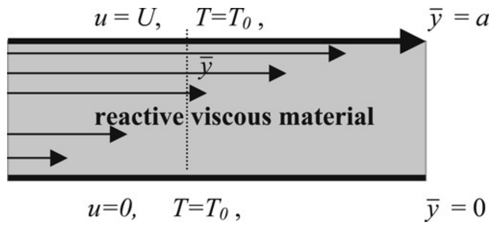

Fig. 1. Geometry of the problem.

conditions can be written as [2,8,9,12]

$$\frac{d^2u}{d\bar{y}^2} = 0, \qquad \frac{d^2T}{d\bar{y}^2} + \frac{QC_0A}{k}e^{-\frac{E}{RT}} + \frac{\mu}{k}\left(\frac{du}{d\bar{y}}\right)^2 = 0, \quad u(a) = U,\ T(a) = T_0,\ u(0) = 0,\ T(0) = T_0, \tag{1}$$

where T is the absolute temperature, U the upper wall characteristic velocity, T_0 the geometry wall temperature, k the thermal conductivity of the material, Q the heat of reaction, A the rate constant, E the activation energy, R the universal gas constant, C_0 the initial concentration of the reactant species, a the channel width, $\bar{y}$ the distance measured in the normal direction and μ the fluid dynamic viscosity coefficient [1–3,5,9]. We introduce the following dimensionless variables into Eq. (1):

$$\begin{aligned} &\theta = \frac{E(T - T_0)}{RT_0^2}, \qquad y = \frac{\bar{y}}{a}, \qquad \lambda = \frac{QEAa^2C_0e^{-\frac{E}{RT_0}}}{T_0^2Rk}, \\ &W = \frac{u}{U}, \qquad \beta = \frac{\mu U^2 e^{\frac{E}{RT_0}}}{QAa^2C_0}, \qquad \varepsilon = \frac{RT_0}{E}, \end{aligned} \tag{2}$$

and obtain the dimensionless governing equation together with the corresponding boundary conditions as (neglecting the bar symbol for clarity)

$$\frac{d^2\theta}{dy^2} + \lambda\left(e^{\left(\frac{\theta}{1+\varepsilon\theta}\right)} + \beta\right) = 0, \qquad \theta(0) = 0, \qquad \theta(1) = 0, \tag{3}$$

where the fluid velocity profile is given as $W(y) = y$ and λ, ε, β represent the Frank Kamenetskii parameter [5], the activation energy parameter and the viscous heating parameter, respectively. In the following sections, Eq. (3) is solved using both perturbation and multivariate series summation techniques [6,7,9–11].

2. Perturbation method

In order to construct an approximate solution to Eq. (3), we employed a regular perturbation method by taking a power series expansion in the Frank Kamenetskii parameter λ, i.e., $\theta = \sum_{i=0}^{\infty} \theta_i \lambda^i$. Substituting the solution series into Eq. (3) and collecting the coefficients of like powers of λ, we obtained and solved the equations governing the coefficients of solution series iteratively. The solution for the temperature field is given as

$$\begin{aligned} \theta(y) = {} & -\frac{\lambda}{2}(\beta + 1)(y^2 - y) + \frac{\lambda^2}{24}(\beta + 1)(y^2 - y)(y^2 - y - 1) \\ & + \frac{\lambda^3}{1440}(\beta + 1)(y^2 - y)(-6y^4\beta + 12y^4\beta\varepsilon + 12y^3\beta - 24y^3\beta\varepsilon \\ & + 6y^2\beta\varepsilon - 3y^2\beta + 6y\beta\varepsilon - 3y\beta + 6\beta\varepsilon - 3\beta + 12y^4\varepsilon - 8y^4 \\ & - 24y^3\varepsilon + 16y^3 + 6y^2\varepsilon + y^2 - 9y + 6y\varepsilon + 6\varepsilon - 9) + O(\lambda^4). \end{aligned} \tag{4}$$

Using a computer symbolic algebra package (MAPLE), we obtained the computer extended series solution up to the first few terms in Eq. (4) as well as the series for the fluid maximum temperature $\theta_{max} = \theta\ (y = 0.5; \lambda, \varepsilon, \beta)$. We are aware that this power series solution is valid for very small Frank Kamenetskii parameter values, i.e. $\lambda \to 0$. However,