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Exact distributions for apparent waves in irregular seas[1]

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Abstract

We discuss the *long-run distributions* of several characteristics for the *apparent waves* in a Gaussian sea. Three types of one-dimensional wave records are considered: 1) the seaway in time at a fixed position; 2) the instantaneous profile along a horizontal line; 3) the encountered seaway. *Exact* integral forms of the *joint* long run distributions are derived for the apparent periods, lengths, and heights. Results of numerical approximations of these distributions are presented in examples. For the computations we considered, as the input spectra, empirical estimates of the frequency spectra as well as JONSWAP type spectra. Effective algorithms are discussed and utilized in the form of a comprehensive computer package of numerical routines. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

1.1. Random sea surface

All recorded wave data exhibit essential irregularity of the sea surface. This irregularity led to the nowadays commonly accepted conviction that any rigorous approach to the seaway has to involve statistical description. The sea surface is then considered

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as a *random* two-dimensional field evolving in time arising as a limit of sums of regular sinusoidal wave trains with random phases. The *directional spectrum* or *spectral density*—an important characteristic for this model—represents half of the squared amplitude of the sinusoidal component in a given direction and with a particular frequency. The spectrum itself, however, does not specify a random field completely, and some further distributional assumptions are needed.

The precise distributional character of the seaway revealed in the field data is rather complicated but Gaussian distributions to a great extent constitute adequate approximations of the empirical ones. In fact, a Gaussian random surface can be obtained as the first order approximation of the solutions to differential equations based on the hydrodynamic theory of deep-water waves. Since the theory of Gaussian random processes is well understood, the assumption about the Gaussian nature of a sea surface equips us with a whole range of powerful theoretical tools which can be applied to statistical analysis. For further discussion of this assumption see also such fundamental works as St Denis and Pierson (1953) and Lewis (1988). In this paper, following numerous other works in the field, we assume the same Gaussian framework as in both the quoted references.

Despite this approximate Gaussian character, there is no doubt that real wave records also exhibit some non-Gaussian features; an important example is the vertical asymmetry characterised by average larger crest heights than trough depths. A physically meaningful approach should involve higher than first order approximations to the Stokes equation. This would lead to the non-linear theory of random sea surfaces which constitutes a research area in its own right. Deriving long run distributions for non-linear, non-Gaussian waves is a theoretically advanced topic which would go beyond the scope of this presentation. Nevertheless, one can circumvent the problem of asymmetry between sizes of crests and troughs by assuming that the surface elevation, although itself not Gaussian, is a smooth transformation of a Gaussian surface. In the literature, there are known parametric forms of either transformations or their inverses (see Ochi and Ahn, 1994) as well as non-parametric methods of their estimation (see Rychlik et al., 1997). Clearly, once we have given the inverse transformation, the approach demonstrated in this work can be applied to the transformed wave records, assuming that they are correctly modeled by a Gaussian process. In Section 4.1, we apply the inverse transformation method to the wave record with evident vertical asymmetry in order to diminish the influence of this non-Gaussian trait.

1.2. Apparent waves

The wave surface is clearly a two-dimensional phenomenon and its study should naturally deal with two-dimensional objects. However theoretical studies of random surfaces still face major difficulties. For example, the evolution in time of the two-dimensional contours of fixed levels—natural objects of study—is highly erratic, often resulting in their merging, splitting or disappearing, and thus very difficult to deal with statistically. Much simpler, yet still meaningful, are studies of one-dimensional records. They can be extracted from a photograph of the sea surface as, for