Gibbs point processes for studying the development of spatial-temporal stochastic processes

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Abstract

Although many studies of marked point processes analyse patterns in terms of purely spatial relationships, in real life spatial structure often develops dynamically through time. Here we use a specific space–time stochastic process to generate such patterns, with the aim of determining purely spatial summary measures from which we can infer underlying generating mechanisms of space–time stochastic processes. We use marked Gibbs processes in the estimation procedure, since these are commonly used models for point patterns with interactions, and can also be chosen to ensure that they possess similar interaction structure to the space–time processes under study. We restrict ourselves to Strauss-type pairwise interaction processes, as these are simple both to construct and interpret. Our analysis not only highlights the way in which Gibbs models are able to capture the interaction structure of the generating process, but it also demonstrates that very few statistical indicators are needed to determine the essence of the process. This contrasts markedly with the relatively large number of estimators usually needed to characterise a process in terms of spectral, autocorrelation or K-function representations. We show that the Strauss-type procedure is robust, i.e. it is not crucial to know the exact process-generating mechanism. Moreover, if we do possess additional information about the true mechanism, then the procedure becomes even more effective. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Although Whittle's (1954) pioneering paper put the study of spatial processes firmly on the statistical map, subsequent developments have mainly centered on purely *spatial* relationships. In real life, however, patterns often develop dynamically through *time*. So although a purely spatial analysis is ideal for achieving a snap-shot summary of spatial structure, such as classifying texture through parameter estimates for a Markov random field, to gain an understanding of the underlying evolutionary processes which generate pattern necessitates a spatial–temporal approach. Bartlett (1975) provided a mechanism for achieving this through the development of the spatial–temporal spectrum, though for some time afterwards there was considerable inertia against conducting space–time statistical analyses. In fairness this was most likely due to the difficulty of collecting appropriate data; modern electronic scanning and video devices have clearly made this far less of a problem.

One of the early biological experiments that provided the impetus for change was performed by Ford (1975), who studied the effect of between-plant competition on *Tagetes patula* L. (marigolds) planted on a regular lattice. A time-lapse film over a three-month period produced a (visual) matrix of continuous-valued variables in continuous time; for the purpose of statistical analysis measurements on plant height were recorded at intervals of 2, 4, 6 and 8 weeks. Ford and Diggle (1981) modelled these data by assuming that competition between individuals was for light. Interaction between neighbours was viewed as a spatial process in relation to differences in plant height, and simulations reproduced both bimodality in the frequency distribution of plant size and an even spatial distribution of large or surviving plants. Renshaw (1984) made theoretical progress by exploiting the underlying lattice structure using a frequency-domain approach. This holds a considerable advantage over space-domain techniques since, unlike sample autocovariances, periodogram values are effectively independent of each other. Further stimulii (Renshaw and Ford, 1983, 1984; Ford and Renshaw, 1984) are provided by the distinctive nature of various ecologically based spectra, and that use of sliding data windows can highlight the presence of nonstationarity.

Development of a theoretical spectrum is greatly helped by the assumption of a linear model. Renshaw (1984) assumes that in the absence of competition plants grow at constant rate λ, and that two plants a distance (r,s) apart compete at rate a_{rs} times their height difference. Then on denoting $\{X_{ij}(t)\}$ to be the set of plant heights at time $t \geq 0$ over $-\infty < i,j < \infty$, we have the linear stochastic equation

$$X_{ij}(t+\mathrm{d}t) = X_{ij}(t)(1+\lambda\,\mathrm{d}t) + \sum_{r,s=-\infty}^{\infty} a_{rs}\{X_{ij}(t) - X_{i+r,j+s}(t)\}\,\mathrm{d}t$$

$$+\mathrm{d}Z_{ij}(t) + \mathrm{o}(\mathrm{d}t), \tag{1.1}$$

where $a_{rs} \geq 0$, $\sum a_{rs} < \infty$ and $\{\mathrm{d}Z_{ij}(t)\}$ is a white-noise sequence with mean 0 and variance $\sigma^2\,\mathrm{d}t$. Although the associated autocovariances can be expressed neatly only in terms of their generating function (a.g.f.), the paradigm between the a.g.f. and the spectrum means that the latter takes a simple form, namely that for