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QUALITATIVE AND QUANTITATIVE ANALYSIS OF STOCHASTIC PROCESSES BASED ON MEASURED DATA, I: THEORY AND APPLICATIONS TO SYNTHETIC DATA

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Analysis of stochastic processes governed by the Langevin equation is discussed. The analysis is based on a general method for non-parametric estimation of deterministic and random terms of the Langevin equation directly from given data. Separate estimation of the terms corresponds to decomposition of process dynamics into deterministic and random components. Such decomposition provides a basis for qualitative and quantitative analysis of process dynamics. In Part I, the following analysis possibilities are described and illustrated using various synthetic datasets: (1) qualitative inspection of the estimated terms presented as fields, (2) reconstruction of the deterministic and stochastic evolution of the process and (3) approximation of the deterministic term by an analytical function and quantitative treatment of the equations obtained. In Part II, these analysis possibilities are applied to experimental datasets from metal cutting and laser-beam welding.

1. INTRODUCTION

In recent decades, processes which generate non-periodic data have been studied intensively. Interest in these processes was fuelled mainly by the theory of deterministic chaos, which showed that non-periodic, even chaotic data can result from a non-linear deterministic process with only a few active degrees of freedom [1, 2]. Numerous analysis methods have been developed to extract meaningful information about the process from its chaotic data [3]. These methods require the data to be generated by a deterministic process, and allow only for negligible measurement noise uncorrelated to the process dynamics. Applicability of these methods to the analysis of data from a stochastic process of which noise is an integral part is limited. However, all experimental data are to some extent noisy, and it is usually difficult to distinguish between noisy chaotic data and stochastic data, which may also be corrupted by measurement noise. The problem is illustrated in Figure 1 using data from a forced oscillator. The phase portrait of the oscillations appears complicated, the time series of the displacement is non-periodic, and the associated power spectrum is broad. Since all these properties are also typical of chaotic data, one might assume that the oscillations are chaotic, and employ the analysis methods inspired by chaos theory [3]. In

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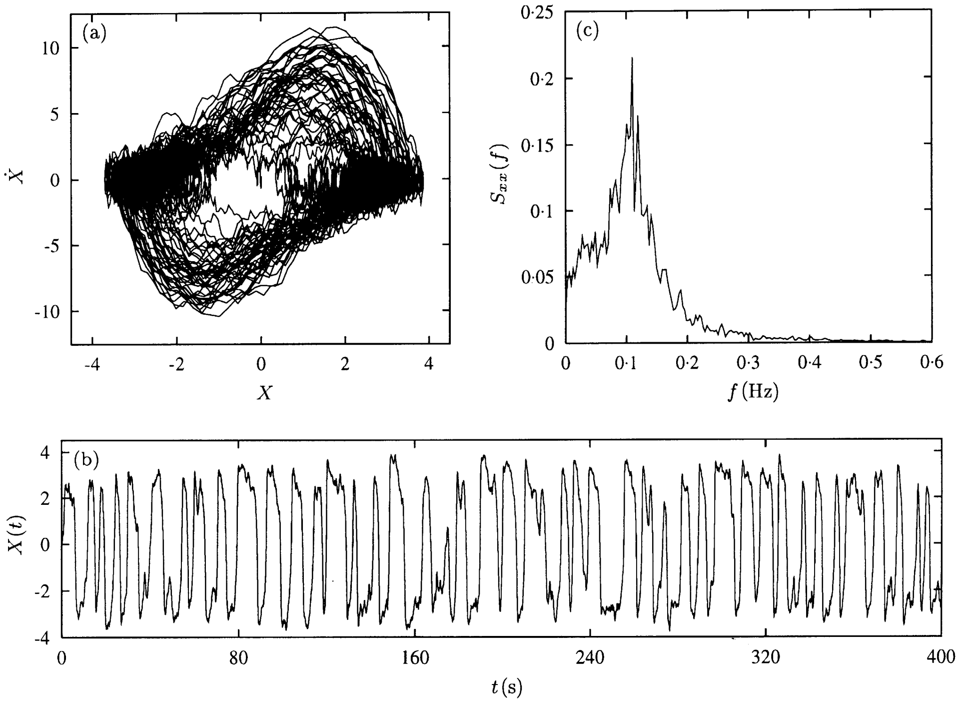

Figure 1. (a) Phase portrait, (b) time series of the displacement and (c) power spectrum of the displacement of a forced oscillator.

the present case, this assumption would be wrong because the data in fact result from a stochastic process, i.e., from the randomly forced van der Pol oscillator in a limit cycle regime (see section A.1 for details). Therefore, methods suited to stochastic data should be employed instead.

The example in Figure 1 belongs to a wide and important class of stochastic dynamic processes that can be described by the Langevin equation:

$$\mathrm{d}/\mathrm{d}t\ (\mathbf{X}(t)) = \mathbf{h}(\mathbf{X}(t)) + \mathbf{g}(\mathbf{X}(t))\boldsymbol{\Gamma}(t). \tag{1}$$

Here $\mathbf{X}(t)$ denotes the time-dependent d-dimensional stochastic variable which characterizes the process state completely. The evolution of $\mathbf{X}$ in time is governed by a sum of a deterministic term $\mathbf{h}$ and a random term $\mathbf{g} \cdot \boldsymbol{\Gamma}$. The random term consists of uncorrelated Gaussian white noise $\boldsymbol{\Gamma}$ and a $(d \times d)$ matrix of noise amplitudes $\mathbf{g}$. No restricting assumptions for $\mathbf{h}$ and $\mathbf{g}$ are necessary. $\mathbf{h}$ can be non-linear which means that deterministic chaos can also be modelled by equation (1). When noise amplitude $\mathbf{g}$ depends on the process state $\mathbf{X}(t)$, noise is of the multiplicative type, whereas constant $\mathbf{g}$ implies the additive type of noise. Note that noise in equation (1) does not affect the process parameters.

A general method for non-parametric estimation of the deterministic and random terms of the Langevin equation (1) has already been proposed [4, 5], and applied to synthetic and experimental datasets from medicine and engineering [6, 7]. The aim of this article is to present several possibilities the method offers for qualitative and quantitative analyses of stochastic data. For this purpose, the method is first reviewed briefly and shows how both the deterministic and random terms of equation (1) can be estimated from data, and inspected qualitatively. Since the terms in fact form a model of the process they can be employed to reconstruct either the deterministic or the stochastic evolution of the process. If equations are needed for the model, the deterministic term can be approximated by an