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Analysis of dynamical systems whose inputs are fuzzy stochastic processes ☆

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Abstract

This paper considers systems whose input signals are fuzzy stochastic processes of second order. The analysis is entirely restricted to discrete time linear time-invariant systems. Convergence conditions of the output are given. The equations on the mean value functions and the covariance functions are derived. The representation of fuzzy stochastic processes is also discussed. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Analysis and design of complex systems often involve two kinds of uncertainty: randomness and fuzziness. The randomness models stochastic variability and fuzziness models measurement imprecision due to linguistic structure or incomplete information. In some fields of application, such as reliability modeling, decision making, data analysis, software reliability and earthquake prediction [2,3,7,10,13,18,19], the uncertainty arises from both randomness and fuzziness simultaneously, and exceeds the realm of the classical probability theory and fuzzy set theory. The concept of fuzzy random variables were introduced by Kwarkernaak [11], Puri and Ralescu [15] to describe fuzzy random quantities. The studies of fuzzy random variables ranged from expectation, limit theorem [8] to martingale [4,16]. For analytical treatment of systems subjected to fuzzy random excitations, Wang and Zhang [17,20] studied the general theory of fuzzy stochastic processes and fuzzy stochastic dynamical systems. However, neither the covariance defined in Ref. [20] is a fuzzy number, nor does it fulfill some main properties of covariance. Recently, Korner [9], Feng et al. [6] defined a real-valued covariance of fuzzy random variables that makes well-defined sense.

In this paper, we will consider a linear time-invariant dynamical system with one input u and one output y. Assuming that the system is characterized by its weighting function h, the input–output relation

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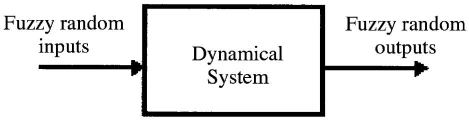

Fig. 1. Fuzzy stochastic system.

can be written as

$$y(t)=\sum_{i=0}^{\infty}h(i)u(t-i). \tag{1}$$

If the input signal u is a fuzzy stochastic process, the response y is also a fuzzy stochastic process (Fig. 1). The problem is then to find the relationships between the fuzzy random properties of the output y and those of the input u. Because of the intrinsical nonlinearity of fuzzy linear algorithms, some conclusions of stochastic systems [1] cannot be directly extended to their fuzzy counterparts.

The present paper is organized as follows. In Section 2, we state some results of fuzzy stochastic processes of second order. In Section 3, conditions for the output to be a fuzzy stochastic process of second order are specified. Then the characteristic equations are derived for fuzzy stochastic systems of nonnegative weighting function, symmetric membership or general case. Illustrative examples are given. Section 4 is dedicated to the representation of fuzzy stochastic processes.

2. Fuzzy stochastic processes of second order

A fuzzy set $u:R\to[0,1]$ is called a *fuzzy number* if it satisfies

(i) u is normal, i.e. $\{x\in R\mid u(x)=1\}$ is nonempty;
(ii) u is fuzzy convex, i.e. $u(\alpha x+(1-\alpha)y)\geqslant \min(u(x),u(y))$ for $x,y\in R,\ \alpha\in[0,1]$;
(iii) u is upper semicontinuous;
(iv) the support set of u is compact, i.e. $\{x\in R\mid u(x)>0\}$ is bounded.

Let F be the set of all fuzzy numbers. The α-cut $u_\alpha=\{x\in R\mid u(x)\geqslant\alpha\}$ of $u\in F$ is a closed interval for any $\alpha\in(0,1]$. The addition and scalar multiplication on F are defined by the following equations:

$$[u+v]_\alpha=u_\alpha+v_\alpha,\quad [\lambda u]_\alpha=\lambda[u]_\alpha,$$
$$u,v\in F,\ \ \lambda\in R,\ \ \alpha\in(0,1].$$

A metric on F is defined by

$$d(u,v)=\frac{1}{2}\int_0^1(|u_\alpha^- - v_\alpha^-|^2+|u_\alpha^+ - v_\alpha^+|^2)\,\mathrm{d}\alpha \tag{2}$$

for all $u,v\in F$, where u_α^-, u_α^+ are the lower and upper endpoints of u_α. (F,d) is a complete metric space according to [6,9]. The norm of a fuzzy number u is defined as

$$\|u\|=d(u,1_{\{0\}}).$$

Let (Ω,A,P) be a complete probability space. A *fuzzy random variable* is a Borel measurable function $X:(\Omega,A)\to(F,d)$. If X is a fuzzy random variable, then X_α^- and X_α^+ are real valued random variables for any $\alpha\in(0,1]$.

Definition 1. The *expectation* of a fuzzy random variable X is defined as a unique $u\in F$ whose α-cuts $u_\alpha=[E(X_\alpha^-),E(X_\alpha^+)]$, $\alpha\in(0,1]$. The real valued *covariance* of two fuzzy random variables X and Y is defined by

$$Cov(X,Y)=\frac{1}{2}\int_0^1(Cov(X_\alpha^-,Y_\alpha^-)+Cov(X_\alpha^+,Y_\alpha^+))\,\mathrm{d}\alpha. \tag{3}$$

The above definition of covariance is consistent with [6,9]. Because for any $\alpha\in(0,1]$, $Cov(X_\alpha^-,Y_\alpha^-)$ is unnecessarily smaller than $Cov(X_\alpha^+,Y_\alpha^+)$, the covariance defined in [20] failed to be a fuzzy number. The *variance* of a fuzzy random variable X can be defined as $Var(X)=Cov(X,X)$.

A fuzzy random variable is said to be of *second order* if $E\|X\|^2<\infty$. It is evident that the expectation and covariance of a fuzzy random variable of second order always exist. Let L_2 be the set of all fuzzy random variables of second order and a metric on L_2 is defined by

$$\rho(X,Y)=\{Ed^2(X,Y)\}^{1/2} \tag{4}$$

for $X,\ Y\in L_2$. We call a sequence $\{X(k),\ k=1,2,\ldots\}$ in L_2 *mean square convergence* to X if $X(k)\xrightarrow{\rho}X$ as $k\to\infty$, and write $X(k)\overset{\text{m.s.}}{\to}X$ or $\lim_{k\to\infty}X(k)=X$.

Lemma 2 (See Ma [14] Theorem 3.1). *If $u\in F$, then*

(i) *u_α^- is a left continuous nondecreasing function on $(0,1]$;*