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Stochastic processes with power-law stability and a crossover in power-law correlations

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Abstract

Motivated by the goal of finding a more accurate description of the empirically observed dynamics of financial fluctuations, we propose a stochastic process that yields three statistical properties: (i) short-range autocorrelations in the index changes, (ii) long-range correlations in the absolute values of the index changes, with a crossover between two power-law regimes at approximately one week, and (iii) power-law stability in the tails of the probability distributions of the index changes. We find that this stochastic process can surprisingly well reproduce statistical properties observed in the high-frequency data of the S&P 500 stock index.

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The recent activity at the interface of statistical physics and economics [1–8] is partly due to the finding that financial data exhibit power-law spatial and temporal scaling behaviors, which are commonly encountered in many different natural phenomena [9]. One common features of those systems is that the power-law spatial or temporal scaling behavior extends over several orders of magnitude. Here, we investigate the possibility that power-law scaling in distributions and correlations may have the same dynamical origin. To exemplify that hypothesis, we study an extensively studied financial time series, the S&P 500 stock index s_t,[1] which has been found to possess the following

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[1] Here Δt denotes the sampling time interval, and we set $\Delta t = 10$ min throughout this paper. We use the S&P 500 data, sampled at 10-min intervals, covering the period 1 January 1984 through 31 December 1995. The length of a trading day is roughly 400 min, and the length of a trading week is 5 days, which corresponds to roughly 2000 min.

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three intriguing statistical features:

(i) The price *changes* defined as $\tilde{r}_t \equiv \log s_{t+\Delta t} - \log s_t$ [1] are short-range correlated [10,11].
(ii) $|\tilde{r}_t|$ are long-range correlated [10,11], and the correlations in $|\tilde{r}_t|$ can be approximated by two piece-wise power laws [11].
(iii) The tails of the probability distributions of the price changes exhibit a stable power-law functional form over a wide range of time scales, called *power-law stability* [12].

In order to develop some understanding of the dynamical origin and of the interrelation of these three statistical features, we propose a stochastic process r_t that is capable of reproducing—qualitatively and quantitatively—the statistical features (i)–(iii) observed in the empirical data $\tilde{r}_t$. Specifically, we define r_t by the following set of coupled equations:

$$r_t = cr_{t-\Delta t} + x_t \, , \tag{1}$$

$$x_t = v_t e_t \, , \tag{2}$$

$$v_t = \sum_{n=1}^{\infty} a_n |x_{t-n\Delta t}| \, . \tag{3}$$

Here, e_t denotes an independent and identically distributed (i.i.d.) random variable with truncated Lévy [13] probability distribution $P(e_t)$,[2] and the *weights* a_n [3] are defined by

$$a_n \sim \begin{cases} n^{-1-\delta_1} & [n < n_\times] \, , \\ n^{-1-\delta_2} & [n \geqslant n_\times] \, , \end{cases} \tag{4}$$

where c, δ_j, and $n_\times$ are four free parameters.[4] The parameter c, which models the short-range correlations, as well as the scaling parameters δ_j and the crossover parameter $n_\times$ can be easily obtained from the data. Note that the values of x_t are not correlated with each other and independent of v_t because e_t are i.i.d. random variables. In contrast, the absolute values of x_t are correlated with each other through the choice of v_t.

The long-range correlations in $|x_t|$ are accomplished through Eqs. (2) and (3), and the specific functional form of the correlations depends on the choice of the weights a_n. If the weights are chosen to decay as a geometric series in n, then the correlations

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[2] We find that the choice of the probability distribution $P(e_t)$ is *not* relevant for the correlation analysis. We find that once the parameters responsible for the correlations in r_t and $|r_t|$ are fixed, the probability distribution for the data can be better approximated by r_t if $P(e_t)$ is chosen to be a truncated Lévy distribution [13] rather than a Gaussian distribution.

[3] Precisely, the weights a_n are defined as $D_j \delta_j \Gamma(n - \delta_j)/(\Gamma(1 - \delta_j) n!)$, where the two constants D_j are set to meet normalization and continuity in the weights a_n. Due to the asymptotic behavior of the Gamma function Γ, the weights a_n can be approximated by $n^{-1-\delta_j}$.

[4] Our choice of a_n is inspired by Ref. [10], which can be understood as a special case of x_t for $\delta_1 = \delta_2$ and $D_1 = D_2 = 1$.