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# Cylindrical tube optimization using response surface method based on stochastic process

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**Abstract**

This paper presents the optimization result in the crashworthiness problem for maximizing absorbing energy of cylindrical tube. To simulate a complicated behavior of this kind of crash problem, a self-developed explicit finite element code is used. The response surface method based on stochastic process is used, that is especially good at modeling the non-linear, multi-modal functions that often bring about in engineering. The main characteristics of using response surface for global optimization lies in balancing the need to exploit the fitting surface for improving the approximation. It can be shown that how these approximating functions can be used to construct an efficient global optimization algorithm. Especially, with the comparison of result by classical optimization method, it can be shown that presented optimization method is independent of noise factor and existence of local minimum.



*Keywords:* Crashworthiness; Response surface method; Stochastic process; Global optimization

## 1. Introduction

In the automotive industry, crashworthiness of a design is of special interest. Non-linear finite element analysis, such as in LS-DYNA [10], PAM CRASH [20], is applied to predict the structural responses. Conclusions from these computations can lead to significant design modifications. It is often hard to determine these design modification from the analysis results. In some cases many variations are tried before a satisfactory design is found. In design considering crashworthiness, only a few algorithms [12] are available to perform sensitivity analysis and optimization as known in linear static analysis.

Tubular structure is very important part in the point of view of crashworthiness, because this kind of structure can reduce the occupant injury in a collision. However, only several attempts [8] have been made to optimize the crashworthiness characteristics of tubular structures. With the implicit finite element analysis, many researchers make an effort to accompany sensitivity analysis for this optimization problem. Nevertheless, there are crucial difficulties for such non-linear sensitivity analysis [9] including noise factor due to contact and large deformation. A ‘noisy’ response may also result in sub-optimal solutions caused by the multitude of local minima. For the explicit finite element method, Yamazaki and Han [8] had gained successful result with general polynomial approximation. This method is very simple and general. But, there are burning questions related with computational efficiency and approximating accuracy.

The response surface method [3,13,17] relies on the fact that the set of designs on which it is based on well selected. However, randomly selected designs may cause an inaccurate surface to be constructed or even prevent the ability to construct a surface at all. For the most simulation case such as sheet metal forming, car crash simulation, etc., simulation is very time-consuming. So, the overall efficiency of the design process depends heavily on the appropriate selection of a design set on experimental design (DOE) is needed. Many experimental design criteria are available but one of the most popular criterion is the D-optimality criterion [5]. In many problems, a function approximation can be constructed on the basis of function values only, and response surface methods based on polynomial functions, or several researchers have studied neural networks. With the use of polynomial interpolation, it becomes difficult to determine the order of the polynomial, and the amount and distribution of data that must be used in developing a suitable response surface. Similar problems exist in the use of neural

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networks. A process of trial and error is typically used in this approach. Furthermore, in the design optimization problems involving non-linear dynamic analysis, there are often 'noisy', the chaotic nature of their response can cause large differences in response to small design changes. In addition, the presence of high frequency noise in the acceleration response of impact or crash problems implies that the unfiltered response becomes almost meaningless.

In this paper, the response surface method based on stochastic modeling process will be adopted for optimization of crashworthiness characteristics of cylindrical tube. The simulation result that have ideal crush deformed pattern without wrinkling is shown. Furthermore, through comparison with the result by classical method, real difficulty due to noise factor could be discussed.

## 2. Response surface method

Response surface method is used for optimization, which was modified and extended the response surface method developed by Schonlau and coworkers [7,14,15] to mechanical non-linear crash problems and prove that superiority. The flow chart of the response surface procedure and optimization are shown in Fig. 1.

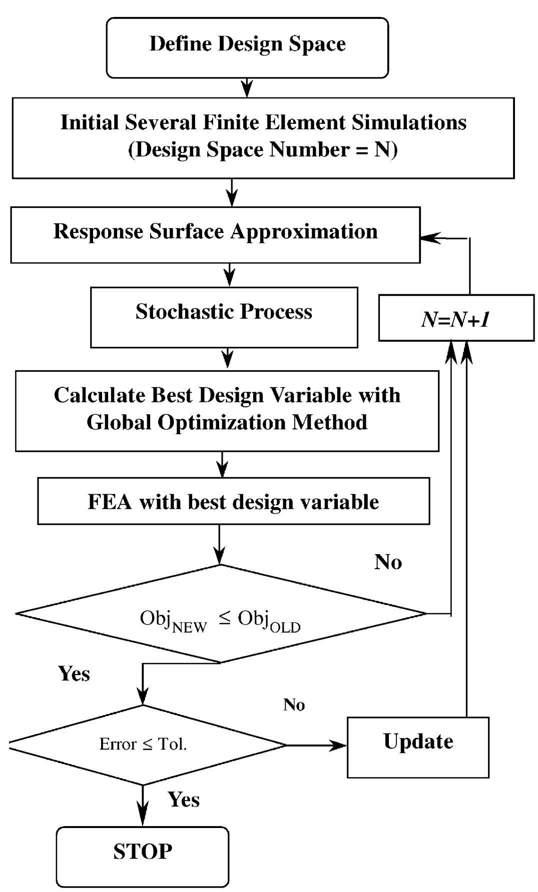


Fig. 1. Flow chart of global optimization for explicit dynamic analysis.

First of all, it is required to explore an approach based on fitting response surfaces to data collected by evaluating the objective and constraint functions at a few points. These response surfaces are then used to visualize input–output relationships, estimate the location of optimum, and suggest points where additional function evaluation may help improve this estimate. The response surface methodology used in this paper is based on fitting the objective and constraint functions with stochastic process [4]. When predicting at a new data point, it is essential to compute the function value that is most consistent with this estimated typical behavior.

This part will focus on using the stochastic process to develop approximating function for where to input search points. These processes balance local and global search in an attractive fashion.

### 2.1. Stochastic process model

Denote the sampled point $i$ by $x^i = (x_1^i, x_2^i, \ldots, x_n^i)$ and the associated function value by $y^i = y(\mathbf{x}^i)$, for $i = 1, 2, \ldots, m$. The simplest and most general response surface function can be expressed as follows:

$$y(\mathbf{x}^i) = \sum_{h=1}^{n} \beta_h f_h(x^i) + \varepsilon^i \quad (i = 1, 2, \ldots, m) \tag{1}$$

In this equation, each $f_h(\mathbf{x})$ is a linear or non-linear function of $\mathbf{x}$. The $\beta_h$'s are unknown coefficients to be estimated. The $\varepsilon^i$'s are normally distributed, independent error terms with mean zero and variance ($S^2$).

In the stochastic process approach, not assuming that the errors are independent, but rather assuming that the correlation between errors is related to the distance between the corresponding points, the special weighted distance formula is used as follows:

$$d_{\mathrm{w}}(x^i, x^j) = \sum_h \theta_h |x_h^i - x_h^j|^{p_h} \quad (\theta_h \geq 0, p_h \in [1, 2]) \tag{2}$$

Using this distance function $d_{\mathrm{w}}$, the correlation between the errors at $x^i$ and $x^j$ is

$$\mathrm{Corr}[\varepsilon(x^i), \varepsilon(x^j)] = \exp[-d_{\mathrm{w}}(x^i, x^j)] \tag{3}$$

It turns out that modeling the correlation in this way is so powerful that the regression terms can be substituted them for a simple constant term:

$$y(x^i) = \mu + \varepsilon(x^i) \quad (i = 1, 2, \ldots, m) \tag{4}$$

where $\mu$ is the mean of the stochastic process, $\varepsilon(x^i)$ the Normal$(0, S^2)$, the correlation between errors is not zero but rather is given by Eqs. (2) and (3). This model has $2n + 2$ parameters ($n$: number of design variables). These parameters will be estimated by choosing them maximize the likelihood of sample. The likelihood function is

$$\frac{1}{(2\pi)^{n/2}(S^2)^{n/2}|\mathbf{R}|^{1/2}} \exp\left[-\frac{(\mathbf{y} - \mathbf{1}\mu)'\mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\mu)}{2S^2}\right] \tag{5}$$