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Production-inventory games: A new class of totally balanced combinatorial optimization games

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Abstract

In this paper we introduce a new class of cooperative games that arise from production-inventory problems. Several agents have to cover their demand over a finite time horizon and shortages are allowed. Each agent has its own unit production, inventory-holding and backlogging cost. Cooperation among agents is given by sharing production processes and warehouse facilities: agents in a coalition produce with the cheapest production cost and store with the cheapest inventory cost. We prove that the resulting cooperative game is totally balanced and the Owen set reduces to a singleton: the Owen point. Based on this type of allocation we find a population monotonic allocation scheme for this class of games. Finally, we point out the relationship of the Owen point with other well-known allocation rules such as the nucleolus and the Shapley value.

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Keywords: Production-inventory games; Totally balanced combinatorial optimization games; Core-allocations; Owen-allocations; Monotonicity rules

1. Introduction

One of the main objectives of management of firms is cost reduction. In order to achieve this goal, groups of firms might form coalitions to diminish operation costs making dynamic

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decisions throughout a finite planning horizon. In tactical planning of enterprises which produce indivisible goods, operation costs mainly consist of production, inventory-holding, and backlogging costs. These coalitions should induce individual and collective cost reductions; thus, stability is achieved in the process of enterprise cooperation.

In our framework a coalition allows each of its members of have access to the technologies owned by the other members of the coalition. Thus, members of a coalition can use the lowest-cost technology of the firms in the coalition. Planning is done throughout a finite time horizon; therefore, at the beginning of each period, the costs to the members of a coalition, which depend on the best technology at that point, may change.

The model that represents that situation is the dynamic, discrete, finite planning horizon production-inventory problem with backlogging. The objective of any group of firms is to satisfy the demand for indivisible goods in each period at a minimum cost. This is a well-known combinatorial optimization problem for which the algorithm by Wagner and Whitin (1958) provides optimal solutions by dynamic programming techniques. The optimal solutions of this problem lead to the best production-inventory policy for the group of firms. These policies generate an optimal operation cost for the entire group. The question is what portion of this cost is to be supported by each firm. Cooperative game theory provides the natural tools for answering this question.

The analysis of inventory situations is not new. Thus, one can find in the literature several centralization inventory models approached from this point of view. The interested reader is referred to Eppen (1979), Kohli and Park (1989), Gerchak and Gupta (1991), Robinson (1993), Hartman and Dror (2003, 2005, 1996), Hartman et al. (2000), Anupindi et al. (2001), Müller et al. (2002), Meca et al. (2003, 2004), Meca (2007), Minner (2007), Tijs et al. (2005) and Slikker et al. (2005) among others, for comprehensive literature on this subject. Other operations research games are studied as well. For a clear and detailed presentation of operations research games (including inventory games) we refer to Borm et al. (2001). We are not aware of any reference in the literature of centralization in inventory models that analyzes inventory models for which optimal operation is only defined implicitly as the optimal solution of a combinatorial problem (as it is the case in the discrete review model in this paper). In this regards, our approach makes a step forward.

The study of cooperative combinatorial optimization games, which are defined through characteristic functions given as optimal values of combinatorial optimization problems, is a fruitful topic (see, for instance, Shapley and Shubik, 1972; Dubey and Shapley, 1984; Granot, 1986; Tamir, 1992; Deng et al., 1999, 2000; Faigle and Kern, 2000). There are characterizations of the total balancedness of several classes of these games. Inventory games and combinatorial optimization games are up to date disjoint classes of games. While in the former class there is always an explicit form for the characteristic function of each game, the characteristic function of the games in the latter class it is defined implicitly as the optimal value of an optimization problem in integer variables.

In this paper we introduce a class of production-inventory games which combine the characteristics of inventory and combinatorial optimization games: this class models cooperation on production and storage of indivisible goods and its characteristic function is defined implicitly as the optimal value of a combinatorial optimization problem. It turns out to be a new class of totally balanced combinatorial optimization games.

We start by introducing definitions and notations in Section 2. In Section 3 we give a complete description of the production-inventory problem (PI-problem). A natural variant of this problem is addressed in Section 4. Several agents, each one facing a PI-problem, decide to cooperate to