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# Inventory performance under pack size constraints and spatially-correlated demand

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ABSTRACT

Motivated by a problem facing a large retailer, we consider the impact of pack size on the performance of a periodic review inventory system in the presence of spatial (i.e., between retailers) correlation of demand—which we model using an equicorrelated multivariate Poisson distribution. Employing simulation, we utilise a full factorial experiment to provide support for decisions on product and supplier selection, and whether or not packs should be split during distribution. We consider variables such as pack size, correlation, and the number of branches, and discuss how they and their interactions impact performance metrics such as inventory and shortage levels and the bullwhip effect.



## 1. Introduction

Improvements in the management of inventories have played a key role in realising gains in supply chain performance. However, theory does not address some of the peculiarities of distribution practice, such as the phenomenon of splitting packs prior to shipment to the retailer.

Our research was motivated by a problem facing a major retailer with almost 100 stores. The distribution centre (DC) receives weekly replenishment requests from each store. The high variance in demand rates between stores had prompted an issue as to whether to split packs prior to the retail level (i.e., at the DC). While this would incur higher labour costs, as well as increased risk of breakage and pilferage, it would reduce inventory costs. There was a concern that the effect of splitting packs on system performance could be complicated by correlation of demand between stores. Little research was found on splitting packs, or spatial correlation, and, to our knowledge, no research has jointly considered these two phenomena.

Seeking to model problems including that described above we evaluate the impact of pack size on the performance of a periodic review inventory system in the presence of spatial correlation (i.e., between retailers) of demand. We consider a two-echelon supply chain consisting of one distributor (or equivalently a DC) and multiple retailers, with retail demands assumed to be from an equicorrelated multivariate Poisson distribution. We employ simulation to assess various performance measures in a wide range of environments using a full factorial experiment.

The paper is structured as follows: the first section summarises literature relevant to the influence of pack size and spatial correlation on inventory performance. The second section outlines the model development, including a description of the transaction logic and demand generation. This third section on experimental analysis

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discusses performance measures, parameter selection, and the simulation. A results section presents an analysis of variance, highlighting the main and interaction effects of the various parameters. We conclude highlighting contributions and potential extensions.

## 2. Literature review

### 2.1. Pack size/minimum order quantity

A key interest of our study is the impact of pack size on supply chain performance. The constraint of pack size, like that of a minimum order quantity (minOQ), results in order batching—a major cause of the bullwhip effect (BWE) (Geary et al., 2006; Lee et al., 1997a; Schultz, 1983; Silver, 1970). Order batching may also introduce demand lumpiness (Geary et al., 2006; Pujawan and Kingsman, 2003), which generates higher average inventory (Kalchschmidt et al., 2003; Pujawan and Kingsman, 2003) and/or reduced customer service levels (Kalchschmidt et al., 2003; Watson, 1987). In addition, Cachon (1999) proves that under a "balanced" ordering strategy, aggregate demand variance increases with retail batch size.

A larger literature has appeared on minOQ policies. Caplin (1985) claims a close positive relationship between the size of individual orders and the variability of aggregate orders in a order-point, order-up-to level *(s, S)* system. Robb and Silver (1998) considering a periodic review, order-up-to level (*R*, *S*) system, argue that the simple practice of rounding recommended order quantities up to the minOQ leads to significant cost penalties in some circumstances, e.g., when the minOQ is much larger than the demand during a review period.

### 2.2. Spatial correlation

A second interest of the study concerns spatial correlation of demand. One can readily argue that such correlation exists (e.g., the demand for charcoal or other heating fuel will rise in two proximal cities during a cold period), and that the variance of the aggregate demand observed by the distributor increases (and the "pooling effect" declines) as spatial correlation increases. However, with some exceptions, inventory research has ignored this phenomenon.

Kelle and Milne (1999) investigate a multi-echelon supply chain in which retailers adopt an (*s*, *S*) ordering policy. They establish the accuracy of approximations for the variance of aggregate purchase orders on the regional supplier, and note that it is an increasing function of the correlation coefficient of retailer demand. Moreover, the ratio between the variance of aggregate purchase orders and the variance of aggregate retail demand is not affected by the correlation of retailer demand.

The paper of Hwarng et al. (2005) indicates that most research on multi-level systems seeks modelling simplicity at the expense of reality and concludes that simplified assumptions on demand distribution lead to significant differences in total costs. They suggest allowing for non-identical retailers with different demand patterns in order to achieve more realistic and reliable results. Key attributes of spatial correlation (i.e., magnitude, sign, and heterogeneity of correlation coefficients) should be considered when modelling the demand distribution.

An empirical study in Yan (2006) analyses one year of sales data (for multiple products and multiple branches) from two organizations—a retail chain and an industrial distributor. It reveals significant, usually positive, spatial correlation exists. However, the magnitude is small (average of 0.04) for the industrial distributor but more prominent (average of 0.34) for the retail chain.

Erkip et al. (1990) detect high spatial correlation (about +0.7) on the same product between branches, caused by sales incentives offered at all locations. They examine the performance of an arborescent two-echelon distribution system where the DC operates only as a cross-docking platform. Assuming fixed replenishment lead times at both the DC and the retailer level, they evaluate the optimal inventory policy of an (*R*, *S*) system in a correlated-demand environment. They demonstrate that the correlation between demands can lead to a significantly higher optimal value of *R*, and indicate the effect of correlations can be so significant that the amplification of standard deviation due to correlation can go as high as twice that of the non-correlated case. They conclude that ignoring such a correlation, especially a high positive correlation, leads to underestimation of the actual standard deviation of demand in the system, and therefore the safety stock level. However, their model does not explicitly separate spatial correlation from autocorrelation (i.e., correlation over time).

Eppen (1979) studies the impact of spatial correlation in a Newsboy context and concludes the pooling effect is more valuable when the branch demands are less positively correlated, assuming a centralised system with multivariate normal demand. The work of Corbett and Rajaram (2006) extends Eppen's finding to the more general near-arbitrary multivariate-dependent demand distributions, considering only pair-wise correlations.

A study by Caron and Marchet (1996), which notes that the correlation coefficient between sales is usually non-negative, investigates the impact of spatial correlation on a two-echelon distribution system, assuming normally-distributed and equicorrelated branch demands. They argue that spatial correlation between several locations results in negligible safety stock savings from pooling. The outcomes reveal that the evaluation of the correlation coefficient is a key factor in the safety stock allocation decision.

## 3. Model development

### 3.1. Transaction logic

The company motivating this research manages, as do many other firms, a two-echelon supply chain with a single DC supplied from vendors, and supplying multiple retailers—which in turn seek to satisfy customer orders from their stock-on-hand (SOH). Backorders