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Periodic forecast and feedback to maintain target inventory level

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ABSTRACT

In cases when a target inventory level is given and the demand is uncertain, it is difficult to provide an appropriate forecast and control policy that ensures a convergence to the inventory target. In this paper two different forecast formulae combined with two different feedback policies are investigated for a periodic review inventory system. A comparison based on analytical and simulation results is presented showing that the form of feedback is critical in case of large demand uncertainty.

1. Introduction

Bonney (1994) and Bonney et al. (1994) surveyed papers and articles on inventory management. It seems that hat theories that had been studied could be classified into some groups by adopting certain criteria. Minimizing (average) inventory cost or maintaining (optimal) inventory level are such criteria.

As an example of studies which are proposed to realize the minimal (average) inventory cost, theories of EOQ-Model are cited. EOQ-Model was first proposed by Arrow et al. (1951). By using this we can determine the ordering points of time and an ordering quantity.

There is another group of studies which are proposed to maintain (optimal) inventory level. Ordering cycles are often determined by some rules of business practice. For such example we can cite the conditions which are presumed by the famous newsboy problem. Assume that a certain inventory level is given as the desired level. If a series of ordering points of time is given, maintaining the appropriate inventory level becomes the main problem. The pioneering study of Simmon (1952) dealt with the case in which a change in the inventory level is described by a differential equation. As an example of a study dealing difference equations, we can cite the work of Vassian (1955). These studies have attempted to apply the results of control theory to inventory control.

2. Presentation of problem

The ordering time points are given. The time interval from the t-th point of time to the $(t+1)$-th point of time is called the t-th ordering period. Further we assume that the lengths of all the ordering periods are equal.

The fundamental equation is given by

$$I(t+1) = I(t) + O(t) - r(t) \tag{1}$$

where $I(t)$ represents the inventory level at the beginning of the t-th period, $O(t)$ represents the order quantity at the beginning of the t-th period, $r(t)$ represents the volume of demand during the t-th period.

$r(t)$ is not known to the policy planner, and $O(^t)$ is given by

$$O(t) = \hat{r}(t) + k \cdot F(I_0,\ I(t), I(t-1), I(t-2), \ldots) \tag{2}$$

where $F(I_0,\ I(t),\ I(t-1),\ I(t-2), \ldots)$ represents a feedback input, k represents feedback gain, $\hat{r}(t)$ represents the forecast of $r(t)$, I_0 represents the desired inventory level.

At the beginning of the t-th period, the exact value of $I(t)$ may not be known to the policy planner. For such a case, the feedback is given by $F(I_0, I(t-1), I(t-2), \ldots)$.

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3. Assumption on demand

We assume that the aggregated demand during any period consists of two parts. One part is expressed by a function $h(t)$ of time t. This part is often referred to as a trend of demand. The other part cannot be expressed by any function of time, but follows certain stochastic rules.

Assumption 1.

$$r(t) = h(t) + Z(t) \tag{3}$$

where $h(t)$ is a known function of t and $Z(t)$ is a random variable satisfying

$$\mathbf{E}[Z(t)] = 0, \quad \mathbf{Cov}(Z(t), Z(t')] = \mathbf{E}[Z(t)Z(t')] = \delta_{t,t'}\sigma^2$$

where $\delta_{t,t'}$ denotes the so-called Kronecker's delta satisfying

$$\begin{aligned} \delta_{t,t'} &= 1 \quad \text{if } t = t' \\ &= 0 \quad \text{if } t \neq t' \end{aligned}$$

Definition 1. For any function $h(t)$, we define $\Delta h(t)$ and $\Delta^2 h(t)$ as

$$\Delta h(t) = h(t+1) - h(t)$$
$$\Delta^2 h(t) = \Delta(\Delta h(t)) = h(t+2) - 2h(t+1) + h(t)$$

Assume $h(t) = b + at$. Now we can show

$$\Delta h(t) = a, \quad \Delta^2 h(t) = 0 \tag{4}$$

Then,

$$\Delta r(t) = a + \Delta Z(t), \quad \Delta^2 r(t) = \Delta^2 Z(t) \tag{5}$$

Moreover from Assumption 1, we can easily derive the following relations:

$$\mathbf{E}[\Delta Z(t)] = 0, \quad \mathbf{V}[\Delta Z(t)] = 2\sigma^2 \tag{6}$$

$$\mathbf{Cov}(\Delta Z(t), \Delta Z(t-1)) = \mathbf{Cov}(Z(t-1), Z(t)) = -\sigma^2 \tag{7}$$

$$m < -1 \quad \text{or} \quad m > 1 \quad \Rightarrow \quad \mathbf{Cov}(\Delta Z(t), \Delta Z(t+m)) = 0 \tag{8}$$

$$\underline{E}[\Delta^2 Z(t)] = 0, \quad \mathbf{V}[\Delta^2 Z(t)] = 6\sigma^2 \tag{9}$$

$$\mathbf{Cov}(\Delta^2 Z(t), \Delta^2 Z(t \pm 2)) = \sigma^2 \tag{10}$$

$$\mathbf{Cov}(\Delta^2 Z(t), \Delta^2 Z(t \pm 1)) = -4\sigma^2 \tag{11}$$

$$m < -2 \quad \text{or} \quad m > 2 \quad \Rightarrow \quad \mathbf{Cov}(\Delta Z(t), Z(t+m)) = 0 \tag{12}$$

For example (10) can be shown in the following ways:

$$\begin{aligned} &\mathbf{Cov}(\Delta^2 Z(t)\ \Delta^2 Z(t+2)) \\ &\quad = \mathbf{E}[Z(t+2) - 2Z(t+1))(Z(t+4) - 2Z(t+3) + Z(t+2)))] \\ &\quad = 2\sigma^2\delta_{t+2,t+4} - 2\sigma^2\delta_{t+1,t+4} + 2\sigma^2\delta_{t,t+4} - 2\sigma^2\delta_{t+2,t+3} \\ &\qquad + 4\sigma^2\delta_{t+1,t+3} - 2\sigma^2\delta_{t,t+3} + \sigma^2\delta_{t+2,t+2} \\ &\qquad - 2\sigma^2\delta_{t+1,t+2} + \sigma^2\delta_{t+2,t} \\ &\quad = \sigma^2\delta_{t+2,t+2} = \sigma^2 \end{aligned}$$

Assumption 2. The initial inventory level is given, and our aim is to plan a policy with which we can maintain this level.

We do not discuss how or why this initial level is given. From the technical viewpoint, this level, which is regarded as the desired (or planned) one, may be predetermined by using a method other than the inventory theory.

4. Expectation of demand level

We collect the data on the series of demands in the past. If the regression method is now applied, a proper is fitted to the time series data of the realized levels of the demand. By extrapolating this curve, we can forecast the demand in the future. However, we are only forecasting demand in the nearest future. We propose two simple forecast formulae.

The first simple formula is

$$\hat{r}(t) = r(t-1) \tag{13}$$

The second finer formula is given by

$$\hat{r}(t) = 2r(t-1) - r(t-2) \tag{14}$$

We will explain the meaning of (14) next.

We can easily show

$$\left.\frac{\mathrm{d}}{\mathrm{d}t} r(t)\right|_{t=t'} = \lim_{-\Delta t \to 0} \frac{r(t' - \Delta t) - r(t')}{-\Delta t}$$
$$\left.\frac{\mathrm{d}}{\mathrm{d}t} r(t)\right|_{t=t'} \approx \frac{r(t' - \Delta t) - r(t')}{{}^{\Delta}t} = \frac{r(t') - r(t' - \Delta t)}{\Delta t}$$
$$r(t') + \Delta t \left.\frac{\mathrm{d}}{\mathrm{d}t} r(t)\right|_{t=t'} \approx r(t') + r(t') - r(t' - \Delta t)$$

It can be easily shown that (14) is obtained by substituting $t' = t - 1$ and $\Delta t = 1$ into above equation.

We generate random demands $r_1(t)$, $r_2(t)$ and $r_3(t)$ with the following formulae:

$$r_1(t) = 50.00 + 1.250 * t + Z_1(t) \tag{15}$$

$$r_2(t) = 50.00 + 50.00 + 0.800 * \mathrm{Exp}(0.029 * t) + Z_2(t) \tag{16}$$

$$r_1(t) = 50.00 + 1.320 * t + 4.50 * \sin(0.50 * \pi t) + Z_3(t) \tag{17}$$

where $Z_1(t)$, $Z_2(t)$ and $Z_3(t)$ are random variables generated in the following ways.

Let $j_i(t)$, $i = 1, 2, 3$ be the value of pseudo-random integers satisfying $0 \leqslant j_i(t) \leqslant 2^{16} - 1$, and $Z_i(t)$ is defined by

$$Z_i(t) = 5.00 * \left(\frac{j_i(t)}{2^{16} - 1} - 0.05\right)$$

The forecast formula $\hat{r}_i(t)$ is defined by

$$\hat{r}_i(t) = 2r_i(t-1) - r_i(t-2), \quad i = 1, 2, 3$$

Table 1 shows the parts of the simulations. From the simulations we obtain

$$r_i(t), \hat{r}_i(t), \quad i = 1, 2, 3; \ \ t = 1, 2, \ldots, 250$$

We can calculate the correlation coefficients between $r_i(t)$ and $\hat{r}_i(t)$, respectively. Five simulations are carried out, and results are shown in Table 2.