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## Risk preferences and robust inventory decisions

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## ABSTRACT

Recently in inventory management instead of maximizing expected profit or minimizing expected cost risk-averse objective functions have been used for determining the optimal order quantity. We use the well-known newsvendor model to determine the optimal order quantity for an objective function with two risk parameters, which can describe risk-neutral, risk-averse as well as risk-taking behaviour of the inventory manager. This approach can also be applied to situations in which the demand distribution cannot be specified uniquely. We consider robust optimization procedures—maximin and minimax regret—to determine optimal order quantities if the set of potential demand variables can be partially ordered by stochastic dominance rules.

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## 1. Introduction

We consider a company, typically a retailer, sourcing a product with short life cycle to stock using the framework of the newsvendor model. Traditionally, risk-neutral inventory managers are considered optimizing the expected profit or cost. But experimental findings state that the actual quantity ordered deviates from the optimal quantity derived from the classical newsvendor model. Only recently, inventory models have been analyzed with objective functions, which do not represent risk neutrality.

Here, we propose a newsvendor model where the objective function is given by a mixture of expected profit and expected shortfall (conditional value at risk). It is a special case of mean deviation rules. Moreover, we consider the newsvendor model under conditions of uncertainty, i.e. the demand distribution is not known exactly; it is assumed that it belongs to a set of distribution functions. Robust inventory decisions take into account the possibility that the distribution of demand may change from the time of ordering to the

start of the selling season (Gallego and Moon, 1993; Brown and Tang, 2006; Perakis and Roels, 2006). We suggest the maximin and the minimax regret approach to handle this kind of uncertainty. Optimal robust decisions minimize the maximal opportunity cost that occurs if the optimal quantity with respect to some demand distribution is not ordered.

In general the optimal robust inventory decisions can only be obtained by simulation. If the demand distributions are stochastically ordered we are able to derive some analytical results. Using our objective function we show that for the risk-neutral and for the risk-averse decision maker the maximin decision is the optimal order quantity corresponding to the dominated demand distribution. Contrary, the optimal robust decision using the minimax regret approach is not uniquely determined.

Thus, the main contributions of the paper are:

- We propose a newsvendor model for risk-averse and risk-taking inventory managers—including also the risk-neutral case—under conditions of uncertainty with respect to the demand distribution.
- Contrary to existing distribution-free newsvendor models we do not assume to know some moments of the demand distribution (e.g. expected value, variance)

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but state that there is a set of potential demand distributions that are stochastically ordered.

- For the maximin approach and the minimax regret approach we derive analytical results concerning the robust order quantity in case of first- and second-order stochastic dominance, respectively.

The paper is organized as follows. In Section 2, we shortly review the classical newsvendor model. In Section 3, we introduce our approach of the newsvendor model. The following sections deal with robust inventory decisions. After a general introduction in Section 4 optimal order quantities are determined when first- and second-order stochastic dominance prevail in Sections 5 and 6, respectively. Section 7 presents conclusions and further developments.

## 2. The classical newsvendor model

In this paper, we concentrate on products in responsive supply chains because products with life cycles shorter than the replenishment leadtime of the supply chain are an increasingly common phenomenon. A prominent inventory model for this framework is the newsvendor model where a retailer has a single opportunity to order the product for the selling season from the supplier. In this model fixed costs are not relevant (Neale et al., 2003, pp. 39–40).

The inventory manager of the retailer has to place the order for a product to the supplier before the actual demand is known. The random demand  $X$  is characterized by the distribution function  $F_X$ . The purchase price per unit of the product is  $c$ . The product is sold to the customers during the regular selling season with a price per unit  $p$ . Unsatisfied demand is lost and leftover inventory of the product at the end of the selling season is sold in another distribution channel with the salvage price per unit  $z$ . Then  $p-c$  describes the cost of understocking by one unit, whereas  $c-z$  describes the cost of overstocking by one unit. It is assumed that  $p > c > z$  holds.

Let  $y$  denote the order quantity and  $g$  denote the profit.  $g$  depends on  $y$  and the stochastic demand  $X$  and is given by

$$g(y, X) = (p - c)y - (p - z)(y - X)^+. \quad (1)$$

In the classical newsvendor model the optimal order quantity  $y_X^*$  is derived by maximizing the expected profit  $E(g(y, X))$ . The optimality condition is given by (see e.g. Chopra and Meindl, 2004)

$$F_X(y_X^*) = (p - c)/(p - z). \quad (2)$$

Therefore, for the optimal order quantity  $y_X^*$  the level of product availability is  $(p-c)/(p-z)$ . The quantile  $(p-c)/(p-z)$  is the cycle service level (CSL); for the order quantity  $y_X^*$  we denote it by  $CSL^*$ .

## 3. The newsvendor model with risk preferences

The newsvendor model with objectives different from expected profit has been an active field of research for a

long time (see Khouja, 1999 for a survey paper). A reason is that experimental findings state that the actual order quantity deviates from the optimal order quantity suggested by the classical newsvendor model (cf. Schweitzer and Cachon, 2000). Also, empirical investigations show that managers tend to order less than the newsvendor because they base their decisions on performance measures other than expected profit (cf. Brown and Tang, 2006).

The seminal paper of the newsvendor model within the expected utility framework is Eeckhoudt et al. (1995). A pragmatic approach for modeling non-risk-neutral inventory managers is proposed by Chen et al. (2004). The impact of an order quantity  $y$  is measured by the conditional value at risk (CVaR) (see (4)).

In inventory models based on concave utility functions or on the CVaR the optimal order quantity is smaller than  $y_X^*$  of the classical newsvendor. Of course the corresponding optimal cycle service level is also smaller than  $CSL^* = (p-c)/(p-z)$ . For example, for the newsvendor model with the CVaR criterion the optimal cycle service level is  $\alpha CSL^*$ , where  $0 < \alpha < 1$  (Chen et al., 2004, p. 7); this model is a special case of our approach (see below).

We propose a newsvendor model that incorporates the experimental and empirical findings: the optimal order quantity can be lower or higher than that of the risk-neutral newsvendor depending on the risk preferences of the inventory manager. The objective function of the model is a convex combination of two conditional expected values of profit: one conditional expected value is the CVaR focusing on low profits whereas the other one focuses on high profits. Formally, in our newsvendor model incorporating risk preferences we use two risk parameters  $\alpha$  and  $\lambda$  to be able to derive optimal order quantities for cycle service levels that can be larger or smaller than the classical newsvendor service level  $CSL^*$ .

The objective is to find the optimal order quantity, which maximizes

$$\Phi(g(y, X)) := \lambda \times E\{g(y, X) | g(y, X) \leq z_\alpha(y)\} + (1 - \lambda) \times E\{g(y, X) | g(y, X) \geq z_\alpha(y)\}, \quad (3)$$

where  $z_\alpha(y)$  is the  $\alpha$ -quantile of the distribution of the random profit  $g(y, X)$  (see (1)),  $z_\alpha(y) = \inf\{z | P(g(y, X) \leq z) \geq \alpha\}$ , with  $0 < \alpha < 1$  and a weighting factor  $\lambda$ ,  $0 \leq \lambda \leq 1$ .

Thus, the objective function is a convex combination of two conditional expected values.

According to the well-known Hurwicz criterion the parameter  $\lambda$  can be interpreted as coefficient of pessimism. Parameter  $\lambda$  can also be derived from exogenously specified performance measures like the cycle service level and the probability of loss (see Jammerneegg and Kischka, 2007, Section 6). The higher  $\lambda$  the more weight the inventory manager puts to low profits. The parameter  $\alpha$  defines the set of low profits for each order quantity  $y$ .

The conditional value at risk  $CVaR_\alpha(g(y, X))$  of the order quantity  $y$  at level  $\alpha$  is given by

$$CVaR_\alpha(g(y, X)) := E\{g(y, X) | g(y, X) \leq z_\alpha(y)\}. \quad (4)$$

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