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Optimal policies for inventory systems with lateral transshipments

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ABSTRACT

Using lateral transshipments can be beneficial in order to improve service levels and to reduce system costs. This paper deals with a single-echelon inventory system with two identical locations. Demands are generated by stationary and independent Poisson processes. In general, if a demand occurs at a location and there is no stock on hand, the demand is assumed to be backordered or lost. However, in this paper, lateral transshipments serve as an emergency supply in case of stock out. In this paper, the rule for lateral transshipments is given, while the ordering policies for normal replenishments are optimized. The transshipment rule is to always transship when there is a shortage at one location and stock on hand at the other. First, we assume that the locations apply (R, Q) policies for normal replenishments, and show that the optimal policies are not necessarily symmetric even though the locations are identical. This means that one cannot in general assume that the optimal policy is symmetric under symmetric assumptions. Second, we relax the assumption of (R, Q) policies and derive the optimal replenishment policy using stochastic dynamic programming.

1. Introduction and literature review

Normally, in divergent inventory systems, installations at some lower echelon replenish stock from a central warehouse at some upper echelon. By introducing the possibility of transshipment of stock between installations at the lower echelon, better service and reduced system cost can be obtained. Consider for instance the case when an installation has no stock on hand when a demand occurs. Then, instead of replenishing stock from the central warehouse with relatively long leadtime, a transshipment from another installation is realized to meet this demand and thus obtain better service. This sort of transshipments is often termed emergency transshipments, since the call for the transshipment originated from a shortage.

Instead of making transshipments when an installation faces a shortage or has no stock on hand, transshipments can be made routinely as a balancing act in order to reduce the system cost. Transshipment policies of this type have often been applied in periodic review models.

Different from most previous literature, we consider a model in which there is a set-up cost for replenishment. In most previous papers it is assumed that set-up costs are negligible, which implies that order-up-to inventory policies are reasonable, see e.g., Axsäter (1990) or Lee (1987). In this paper we derive the optimal replenishment policy under a given transshipment rule, but also examines how well the commonly used (R, Q) policy performs compared to the optimal policy.

One of the first papers which mentioned the transshipment problem was Clark and Scarf (1960). However, they ignored the problem due to the mathematical complexity. In another early paper, Krishnan and Rao (1965) develop a periodic review, single-echelon model in which they allow transshipments between the lower echelon stock facilities. Gross (1963) considers a transshipment model where it is assumed that both ordering and transshipments are made before demand is realized.

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Some papers which present results on optimality are Das (1975), Robinson (1990), Archibald et al. (1997), Herer and Rashit (1999), and Axsäter (2003).

Das (1975) considers a two-location, single-echelon, and single-period problem with periodic review. The main contribution of Das is the opportunity to transfer stock in the middle of a period. He derives optimal transfer and ordering rules by using stochastic dynamic programming, and shows that the so-called *complete pooling* policy is optimal.

Robinson (1990) studies a multi-period and multi-location problem where transshipments between locations are possible. Under the assumptions of negligible transshipment- and replenishment leadtimes, he demonstrates the optimality of the order-up-to policy. However, analytical results are only found in the two-location case. In the general case, Robinson suggests a heuristic solution method.

Archibald et al. (1997) examine an inventory system much related to Robinson (1990). One difference is that in Archibald et al. transshipments can be made at any time during the period. They formulate the problem as a Markov decision process, and allow emergency orders from the external supplier if a transshipment from another location is not possible. Also in this paper all leadtimes are assumed to be zero. Clearly, it is a major limitation to assume that replenishments occur instantaneously since transshipment and replenishment policies in general depend on both stock on hand and orders in transit.

Herer and Rashit (1999) study transshipment with fixed and joint replenishment costs, but only with single period.

A quite recent paper by Minner and Silver (2005) considers a distribution system with two identical locations, in which lateral transshipments are allowed. The rule for lateral transshipments is, however, not optimized. The locations apply (R, Q) policies, and demand occurs according to a compound Poisson process. They assume that all unsatisfied demand after transshipments is lost, and develop heuristics in order to being able to evaluate costs.

2. Problem formulation

In this paper we consider a single-echelon system with two parallel locations. Demands are generated by stationary Poisson processes, and the delivery leadtimes for the locations follow an exponential distribution. We assume that the time for a transshipment is negligible and therefore set to zero. Normally it is assumed that all unsatisfied demands are backordered or lost. However, throughout this paper, if a demand occurs at a location and there is no stock on hand, a transshipment from the other location is realized if the other location has stock on hand. If this is not possible the demand is lost.

Let us list the following basic notation:

L	leadtime for normal replenishments,
λ	customer arrival intensity,
γ	arrival intensity for outstanding orders,
R_i	reorder point at location i,
Q_i	batch quantity at location i,
h	holding cost per unit and time unit,
ℓ	lost sales cost per unsatisfied demand,
A	ordering cost per batch,
τ	transshipment cost per unit.

Since the locations are identical we have no subscript i in the notations except for R_i and Q_i (because R_i and Q_i shall be optimized).

The main purpose of this paper is to derive optimal replenishment policies for the locations when lateral transshipments are allowed in the model. The transshipments are carried out according to the given simple transshipment rule, as described above. This means that we will not consider optimal transshipment policies. Our second aim is to show that in some cases with identical locations, it is optimal to let only one location replenish stock from the supplier, and let the other location replenish stock via lateral transshipments. This means that we arrive at an asymmetric policy under symmetric assumptions. In Section 3, we assume that the locations are identical and apply continuous review (R, Q) policies. However, in Section 4 we shall instead derive the optimal replenishment policy by stochastic dynamic programming.

3. A transshipment model with (R, Q) policies

A very common problem in inventory theory is to determine the optimal reorder point and order quantity in order to minimize some cost function. We consider a single-echelon system with two parallel locations, as shown in Fig. 1. We assume that the locations are identical and apply continuous review (R, Q) policies. If a transshipment is realized, the sending location triggers a replenishment order from the supplier if the inventory position, *IP* (stock on hand + outstanding orders), drops to R. If it is not possible to make a transshipment (this occurs when a customer arrives at some location, and neither location 1 or 2 has stock on hand), the demand is assumed to be lost. Batch ordering models without lateral transshipments and with lost sales are already quite complicated, see e.g., Rosling (2002) or Johansen and Thorstenson (1996). Evidently, batch ordering models with lost sales which

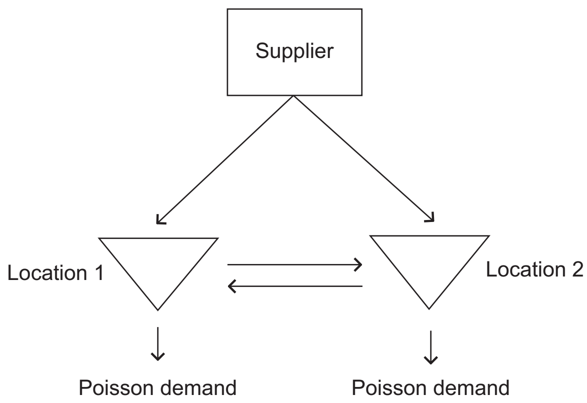

Fig. 1. Two identical locations, which apply (R, Q) policies.