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A generalized algebraic model for optimizing inventory decisions in a multi-stage complex supply chain

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ABSTRACT

In this paper, we deal with more generalized inventory coordination mechanism in an n-stage, multi-customer, non-serial supply chain, where we extend and generalize pervious works that use algebraic methods to optimize this coordinated supply chain. We establish the recursive expressions for this multi-variable optimization problem. These expressions are used for the derivation of the optimal replenishment policy and the development of the solution algorithm. Further, we describe a simple procedure that can help in sharing the coordination cost benefits to induce all stages to adopt the inventory coordination mechanism. We provide a numerical example for illustrative purposes.

1. Introduction

Supply chain management can be defined as a set of approaches utilized to efficiently integrate suppliers, manufactures, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide cost while satisfying service level requirements (Simchi-Levi et al., 2003).

In the past, supply chain production-inventory decisions were not coordinated among the different parties in the supply chain. This lack of coordination leads to weakly connected activities and decisions across the supply chain. Recently firms realized that the global system performance and cost efficiency can be improved through closer collaboration among the chain partners and through high level of coordination of various decision processes. Supply chain inventory-distribution coordination can be achieved by coordinating the cycle time across the chain stages. The simplest way of cycle coordination is the equal cycle under which the same cycle is followed throughout the supply chain. But many supply models achieve coordination by following the integer multipliers mechanism in which the cycle time at each stage is an integer multiple of the cycle time of the adjacent downstream stage, or by integer powers of two multipliers at each stage mechanism (Khouja, 2003).

In recent years numerous articles in supply chain modeling have addressed the issue of inventory coordination. Banerjee (1986) introduced the concept of joint economic lot sizing problem (JELS). He considered the case of a single vendor and a single purchaser under the assumption of deterministic demand and lot for lot policy. Goyal and Szendrovits (1986) presented a constant lot size model where the lot is produced through a fixed sequence of manufacturing stages, with a single setup and without interruption at each stage. This model mainly, relaxes the constraint that batches must be of equal size at any particular stage. Goyal (1988) provided a more general model for the case of single vendor single buyer through relaxing the lot-for-lot policy. He showed that his model provides a lower or equal total joint relevant cost compared to Banerjee

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(1986). Goyal and Gupta (1989) extensively reviewed the literature which deals with the interaction between a buyer and vendor. They classified the literature dealing with the integrated models into four main classes. The first class represents models which deal with joint economic lot sizing policies. The second class characterizes models which deal with the coordination of inventory by simultaneously determining the order quantity for the buyer and the vendor. The third class is a group of models which deal with integrated problem but do not determine simultaneously the order quantity of the buyer and the vendor. The last class represents models which deal with buyer vendor coordination subject to marketing considerations. Lu (1995) developed a one-vendor multi-buyer integrated inventory model with the objective of minimizing the vendor's total annual cost subject to the maximum cost that the buyer may be prepared to incur. Goyal (1995) revisited the single-single-buyer where he relaxed the constraint of equal sized shipments of Goyal (1988) and suggested that the shipment size should grow geometrically.

Hoque and Goyal (2000) extended the idea of producing a single product in a multistage serial production system with equal and unequal sized batch shipments between stages. Khouja (2003) considered the case a three-stage non-serial supply chain and developed the model to deal with three inventory coordination mechanisms between the chain members. Ben-daya and Al-Nassar (2008) relaxed the assumption of Khouja (2003). Regarding the completion of the whole production lot before making shipments out of it and assumed that equal sized shipments take place as soon as they are produced and there is no need to wait until a whole lot is produced.

The use of differential calculus to model the integrated production inventory systems is common in the area of operational research. However, several researchers focused on the easy solution methods for the optimization of these types of systems. Cárdenas-Barrón (2001) used algebraic procedure to the EPQ formula taking shortages into consideration within the case of only one backlog cost per unit and time unit.

Cárdenas-Barrón (2007) formulated and solved an n-stage-multi-customer supply chain inventory model where there is a company that can supply products to several customers. The production and demand rates were assumed constant and known. This model was formulated for the simplest inventory coordination mechanism which is referred to as the same cycle time for all companies in the supply chain. It was concluded that it is possible to use an algebraic approach to optimize the supply chain model without the use of differential calculus. Chung and Wee (2007) considered an integrated three-stage inventory system with backorders. They formulated the problem to derive the replenishment policies with four-decision-variables algebraically. Wee and Chung (2007) also used a simple algebraic method to solve the economic lot size of the integrated – buyer inventory problem. As a result, students who are unfamiliar with calculus may be able to understand the solution procedure with ease. Chi (2008) presented a simple algebraic method to demonstrate that the lot size solution and the optimal production-inventory cost of an imperfect EMQ model can be derived without derivatives. Kit-Nam Francis Leung (2008) considered the EOQ problem, where the quantity backordered and the quantity received are both uncertain. He used the complete squares method to derive a global optimal expression from a non-convex objective function in an algebraic manner. Cárdenas-Barrón (in press) considered the problem of optimal manufacturing batch size with rework process at single-stag production system. He determined the optimal solution for two different inventory policies. He also established the range of real values of proportion of defectives products for which there is an optimal solution, the closed-form for the total inventory cost for both policies, the mathematical expressions for determining the cost penalty and the additional total cost for working with a non-optimal solution.

In this study, we develop an optimal replenishment policy using a simple algebraic method to solve the n-stage, multi-customer, non-serial supply chain inventory problem. Our work is an extension of the three stage supply chain model in Khouja (2003). We consider the integer multiplier coordination mechanism.

The remainder of this paper is organized as follows. The next section presents the notation and assumptions made in the model. Section 3 describes the development of the model. A solution procedure is presented in Section 4. Section 5 describes a scheme for sharing the coordination benefits. Section 6 presents a numerical example. Finally, Section 7 contains some concluding remarks.

2. Notation and assumptions

The following notations are used in developing the model:

T = Basic cycle time, cycle time at the end retailers.
T_i = Cycle time at stage i.
S_{ij} = Setup cost for firm j at stage i.
S_i = Total setup cost for all firms at stage i.
K_i = Integer multiplier at stage i.
h_i = Inventory holding cost at stage i.
n_i = Number of firms at stage i.
D_{ij} = The mean demand rate of firm j at stage i.
P_{ij} = Production rate of firm j at stage i.
A = The product of all production rates for all the companies in the supply chain.
B_{ij} = The product of all production rates for all the companies in the supply chain, except for the company j in stage i.