Transportation Research Part E 45 (2009) 572–582

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

A technical note on "Optimizing inventory decisions in a multi-stage multi-customer supply chain"

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ARTICLE INFO

Article history:
Received 9 September 2008
Received in revised form 4 January 2009
Accepted 24 January 2009

Keywords:
Inventory
Production
Without derivatives
The perfect squares method

ABSTRACT

We first generalize Khouja [Khouja, M., 2003. Optimizing inventory decisions in a multi-stage multi-customer supply chain. Transportation Research Part E: Logistics and Transportation Review 39 (3), 193–208] integrated model considering the integer multipliers mechanism and next individually derive the optimal solution to the three- and four-stage model using the perfect squares method, which is a simple algebraic approach so that ordinary readers unfamiliar with differential calculus can understand the optimal solution procedure with ease. We subsequently deduce the optimal expressions for Khouja (2003) and Cárdenas-Barrón [Cárdenas-Barrón, L.E., 2007. Optimal inventory decisions in a multi-stage multi-customer supply chain: a note. Transportation Research Part E: Logistics and Transportation Review 43 (5), 647–654] model, and identify the associated errors in Khouja (2003). We present two numerical examples for illustrative purposes. We finally shed light on some future research by extending or modifying the generalized model.

1. Introduction

Increasing attention has been given to the management of a multi-stage multi-firm (or multi-customer) supply chain in recent years. This is due to increasing competitiveness, short life cycles of modern electronic products and the quick global changes in today's businesses. The integration of the supply chain provides a key to successful international business operations. This is because the integrated approach improves the global system performance and cost effectiveness. Besides integrating all members in a supply chain, to improve the traditional method of solving inventory problems is also necessary. Without using derivatives, Grubbström (1995) first derived the optimal expressions for the classical economic order quantity (EOQ) model using the unity decomposition method, which is an algebraic approach. Adopting this method, Grubbström and Erdem (1999) and Cárdenas-Barrón (2001), respectively, derived the optimal expressions for an EOQ and economic production quantity (EPQ) model with complete backorders. In this note, a generalized model for a three- or four-stage multi-firm production-inventory integrated system is solved using the revised version of the perfect squares method, which is also an algebraic approach; whereby optimal expressions of decision variables and the objective function are derived.

In addition to the papers with regard to solving some inventory models without derivatives surveyed by and classified in Table 1 of Cárdenas-Barrón (2007), we review some recently relevant papers as follows: using the unity decomposition method, Chiu et al. (2006) derived the optimal expressions for an EPQ model with complete backorders, a random proportion of defectives, and an immediate imperfect rework process while Cárdenas-Barrón (2008) derived those for an EPQ model with no shortages, a fixed proportion of defectives, and an immediate or a *N*-cycle perfect rework process. Using the complete squares method and perfect squares method proposed by Chang et al. (2005), Wee and Chung (2007) and Chung and

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doi:10.1016/j.tre.2009.01.007

Wee (2007), respectively, derived the optimal expressions for a two- and three-stage single-firm supply chain inventory model with complete backorders, and lot streaming (which means that any shipments can be made from a production batch before the whole batch is finished). Leung (2008a) proposed revised versions of the complete and perfect squares methods to derive the optimal expressions for an EOQ model with partial backorders and Leung (2008b) also adopted them to derive those for an EOQ model when the quantity backordered and the quantity received are both uncertain. Teng (2008) proposed the arithmetic–geometric-mean-inequality method to derive the optimal expressions for the classical EOQ model. Wee et al. (2009) proposed a modified version of the cost-difference comparisons method originated from Minner (2007) to individually derive the optimal expressions for an EOQ and EPQ model with complete backorders.

2. Assumptions and notation

Our multi-stage multi-firm supply chain inventory-production model is based on the assumptions stated by Khouja (2003), with the following five main exceptions:

(1) The setup or ordering costs are different for all firms in the chain.
(2) The holding costs of raw materials are different from those of finished products.
(3) The holding costs of raw materials are different for all firms in the chain.
(4) The holding costs of finished goods are different for all firms in the chain.
(5) There are three or more stages.

We thus generalize Khouja's (2003) model by incorporating these five realistic conditions. The following notation (almost all as defined in Khouja, 2003) is used in the expression of the joint total relevant cost per year.

D_{ij} = demand rate of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n)$ [units per year]
P_{ij} = production rate of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ [units per year]
S_{ij} = setup or ordering cost of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n)$ [$ per cycle]
$g_{ij} \equiv h^{(rm)}_{i-1,j}$ = holding cost of incoming raw material of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ [$ per unit per year]
h_{ij} = holding cost of finished goods of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n)$ [$ per unit per year]
$T_{nj} = T$ = basic cycle time of firm $j(=1,\ldots,J_n)$ in stage n (T is a decision variable with *non-negative* real values) [a fraction of a year]
$T_{ij} = T\prod_{k=i}^{n-1}K_k$ = integer–multiplier cycle time of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ ($K_1,\ldots,K_{n-1}$ are decision variables, each with *positive* integral values) [a fraction of a year]
TC_{ij} = total relevant cost of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n)$ [$ per year]
$JTC(K_1,\ldots,K_{n-1},T) = \sum_{i=1}^{n}\sum_{j=1}^{J_i}TC_{ij}$ = joint total relevant cost as a function of $K_1,\ldots,K_{n-1}$ and T (the objective function) [$ per year]

To simplify the presentation of the subsequent mathematical expressions, we designate

$$\varphi_{ij} = \frac{D_{ij}}{P_{ij}} \quad \text{for } i = 1,\ldots,n-1;\ j = 1,\ldots,J_i, \tag{1}$$

$$S_{i\cdot} = \sum_{j=1}^{J_i} S_{ij} \quad \text{for } i = 1,\ldots,n, \tag{2}$$

$$H_1 = \sum_{j=1}^{J_1} D_{1j}[\varphi_{1j}(g_{1j}+h_{1j})+h_{1j}], \tag{3}$$

$$H_i = \sum_{j=1}^{J_i} D_{ij}[\varphi_{ij}(g_{ij}+h_{ij})+h_{ij}] - \sum_{j=1}^{J_{i-1}} D_{i-1,j}h_{i-1,j} \quad \text{for } i = 2,\ldots,n-1, \tag{4}$$

and

$$H_n = \sum_{j=1}^{J_n} D_{nj}h_{nj} - \sum_{j=1}^{J_{n-1}} D_{n-1,j}h_{n-1,j}. \tag{5}$$

The total relevant cost per year of firm $j(=1,\ldots,J_i)$ in stage $i(=1,\ldots,n-1)$ is given by

$$TC_{ij} = \frac{\prod_{k=i}^{n-1}K_k \cdot TD_{ij}^2}{2P_{ij}}\cdot(g_{ij}+h_{ij}) + \frac{(\prod_{k=i}^{n-1}K_k - \prod_{k=i+1}^{n-1}K_k)TD_{ij}}{2}\cdot h_{ij} + \frac{S_{ij}}{\prod_{k=i}^{n-1}K_k\cdot T}, \tag{6}$$

where term 1 represents the sum of the holding cost of raw material as it is being converted into finished goods and that of finished goods during the production portion of a cycle, term 2 represents the holding cost of finished goods during the non-production portion of a cycle, and term 3 represents the setup cost.