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A perishable inventory model with Markovian renewal demands

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ABSTRACT

In the inventory model, people usually assume that the inter-demand time is independently identical distributed which may not be true in reality. Here we study an (s, S) continuous review model for items with an exponential random lifetime and a general Markovian renewal demand process. By constructing Markovian renewal equations, we derive the mean and the variance of the reorder cycle time and lead to a simple expression for the total expected long run cost rate. The numerical results illustrate the system behavior and lead to managerial insights into controlling such inventory systems.

1. Introduction

In this paper, we study a continuous review model for items with an exponential random lifetime and a Markovian renewal demand process in which the inter-demand time is generally distributed. Perishability is a wide spread phenomena existing in many sectors. For example, in supermarkets, the fresh food may deteriorate gradually before it should get consumed; the electronic products may age while still in storage; fashion may become out of date when the seasons change. These perishable goods seize a large proportion of inventory so that the ordering policies determined by the conventional inventory models are not appropriate, so it requires building up a particular perishable inventory model to study the optimal ordering policy.

The former perishable inventory literature that deals with random lifetimes has only been limited to some simple models. Kalpakam and Arivarignan (1988) studied a continuous review (s, S) model with Poisson demand and an exponential lifetime. By assuming a zero lead time and no backorder for perishable inventory, the authors concluded that the reorder point s should be set to 0. Liu (1990) studied a similar continuous review (s, S) model with backorders and the reorder point s is suggested to be smaller than or equal to 1. Kalpakam and Sapna (1994) extended Kalpakam and Arivarignan's (1988) results to the case with the exponentially distributed lead time and only one outstanding replenishment order. Liu and Yang (1999) considered the exponential replenishment lead time but no restriction the number of outstanding replenishment orders. Further more, Liu and Shi (1999) provided a comprehensive treatment of the model in Liu (1990) with a general renewal demand process. The authors stated a simple but important relation which relates two important system performance measures, the expected inventory level and the expected length of the cycle time between two reorder epochs. By means of supplementary variable method, the authors constructed a Markov process so as to obtain the expected length of the cycle time.

Classical inventory models have assumed that the inter-demand times are identically independent distributed (see Nahmias's, 1982 review paper). As elaborated in Song and Zipkin (1993), many randomly changing environmental factors, such as fluctuating economic conditions and uncertain market conditions in different stages of a product life-cycle, can have a major effect on demand. For such situations, the Markov approach provides a natural and flexible alternative for modeling the demand process (Sethi and Cheng, 1997). When the product has a random

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lifetime, the modeling and optimization of inventory systems with Markovian demand is more difficult.

We here extend Liu and Shi's (1999) work by considering a general Markovian renewal demand process and developing closed-form results for the case with backordering. We contribute to the literature not only by providing the first perishable inventory model with Markovian renewal demand, but also presenting an approach to obtain the expected length of the reorder cycle time. The supplementary variable method used in Liu and Shi (1999) may not be suitable here because a complicated system of differential equations would be involved. Since the epochs of demand arrivals are Markovian renewal points, by introducing a Markovian renewal process, we can easily derive some recursive equations so as to calculate the cycle time. Although the assumptions of zero lead time and exponential lifetime may not be realistic, the approach of Markovian renewal process used in this paper provides us a way to study some more complicated and interesting models.

We now present a brief review on other related works. When the lifetime of items is fixed, the periodic review model with zero lead time was investigated by Fries (1975) and Nahmias (1975b). Later efforts have been largely focused on finding and computing approximations of the true optimal policy (Chazan and Gal, 1977; Cohen, 1976; Nahmias, 1975a, 1976, 1978; Nandakumar and Morton, 1993). Weiss (1980) studied a continuous review perishable inventory model with a Poisson demand process, fixed lifetime and zero lead time. Ravichandran (1995) analyzed a non-standard perishable model with a positive random lead time and a Poisson demand process. Chiu (1995a, b) developed an approximation for the expected outdating of the current order Q. Further, using the Markov renewal theory, Liu and Lian (1999) and Lian and Liu (2001) analyzed the structure of the cost function and derive the optimal reorder point and order-up-to level when the demand is a renewal process with random batch size. Lian and Liu (1999) and Lian et al. (2005) obtained the closed form of the cost function in a discrete time inventory model with finite lifetime.

The remainder of this paper is organized as follows. In Section 2, we define the cost structure and identify the format of the cost function. In Section 3, we present the analytical results. We first define a Markovian renewal process and construct the transition matrix of the embedded Markov chain, then derive the recursive equations to calculate the length of the reorder cycle and deduce the closed-form expressions for the cost elements called for in the cost function given in Section 2. Section 4 presents the numerical results to demonstrate how the optimal policy of the model with the Markovian renewal demand is different from the model with the renewal demand. We conclude the paper in Section 5.

2. Model assumption and definition

We consider the inventory replenishment problem for one product. The lifetime X of the unit in-stock is assumed to be exponentially distributed with a constant failure rate λ. As mentioned in Liu and Shi (1999), exponential time-to-failure has been widely adopted in reliability and production models for machines and components in operation. For stand-by items and items in storage, aging, failure, deterioration, or spoilage are widespread in manufacturing firms, warehouses, defense hardware storage, and retail outlets. We adopt the exponential lifetime assumption so that we can capture the randomness of the lifetime and focus on the impact of different demand processes.

We define a stochastic process $\{D(t), t \geq 0\}$ with state space $E = \{1, 2, \ldots, m\}$ where $D(t)$ is defined as the demand state at time t. Let t_n be the epoch of the nth demand arrival ($t_0 = 0$), $n = 1, 2, \ldots$, then $D_n \hat{=} D(t_n)$ is the demand state at the epoch of the nth demand arrival. We assume that $\{D_n, t_n, n \geq 0\}$ is a time-homogeneous Markovian arrival process (see Cinlar, 1975), i.e., for any integer $n \geq 0$, $\forall d_0, \ldots, d_{n-1}$, $k, r \in E$, and $\forall t \geq 0$,

$$\begin{aligned} P\{D_{n+1} = j, t_{n+1} - t_n \leq t | D_k = d_k; k = 0, \ldots, n-1,\\ D_n = i; t_0, \ldots, t_n\} \\ = P\{D_{n+1} = j, t_{n+1} - t_n \leq t | D_n = i\} \\ = G_{i,j}(t). \end{aligned} \tag{1}$$

We call the transition probability matrix $G(t) = (G_{i,j}(t))_{m\times m}$ semi-Markov kernel over E. The matrix $G = G(+\infty)$ is an irreducible stochastic matrix. And $\pi = (\pi_1, \ldots, \pi_m)$ is the stationary probability vector of the transition probability matrix G.

We assume that each arrival demand only requests one unit of item. All unmet demands are backordered.

The system state is reviewed continuously. Following Weiss (1980), it can be shown that the optimal replenishment policy for this model is of the (s, S) type when backorders are allowed and the replenishment lead time is zero. We here focus on finding the optimal values of s and S. Whenever the inventory level $I(t)$ drops to the reorder point s, an order of size $S - s$ is placed. Since the lifetime is finite and random, the actual inventory level may decrease at any time due to failures.

We assume that delivery of a replenishment order is immediate. We note that ordering up to $S = 0$ is always better than ordering up to $S < 0$ because it saves shortage cost, ordering cost and does not incur any additional inventory cost. We also note that with the (s, S) continuous review policy and zero lead time, the inventory level satisfies the following inequality, $s + 1 \leq I(t) \leq S$, where $I(t)$ is the inventory level at time t. Furthermore, as the lead time equals to zero, if s is set to be -1, the last demand batch before the replenishment order will be satisfied immediately without delay. Hence, theoretically, $s = -1$ is always a better reorder point than $s \geq 0$ (see Weiss, 1980; Lian and Liu, 2001). For example, assume that the order quantity is one unit, we need to pay the holding cost for a period between the epoch when the order arrives and the epoch when a demand arrives if we make an order when $s = 0$, but we do not need to pay either holding cost or shortage cost if you make an order when $s = -1$. Therefore, we assume that $s \leq -1$, and such, the entire stock is renewed each time and order is placed.