

# A single-item inventory model for expected inventory order crossovers

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## Abstract

Expected inventory order crossovers occur if at the moment of ordering it is expected that orders will not arrive in the sequence they are ordered. Recent research has shown that (a) expected inventory order crossovers will be encountered more frequently in future, and that (b) use of a myopic order-up-to policy based on a stochastic dynamic programming approach leads to improved performance compared to the classical approach. In this paper, we show that the improved policy is still heuristic in nature, as it neglects several control options that are available on the various ordering moments and makes some restrictive assumptions with respect to the separability (i.e., decomposability) of the stochastic dynamic programming problem. We propose further improvements in the policy for situations where a quadratic cost function is appropriate.

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*Keywords:* Inventory management; Control policy; Mathematical modeling

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## 1. Introduction

Order crossovers occur if orders do not arrive in the same sequence as they were ordered. This phenomenon is caused by differences in the lead-times and review times of the orders. Order crossovers are often neglected in inventory models. Whenever they are taken into account, most inventory models assume that lead-time differences are caused by a stochastic process (Fig. 1). This paper focuses on order crossovers that are the

result of dynamic processes instead of stochastic processes.

In dynamic processes, lead-time fluctuations that occur are known in advance and can be anticipated upon. Dynamic lead-time fluctuations may occur due to contract changes, expediting policies, dual-sourcing policies from different geographical areas, transportation mode changes, etc. Riezebos (2006) has argued that these dynamic variations will be encountered more frequently in future. The main reason is that the above-mentioned instruments are increasingly used in modern supply chain management in order to increase flexibility (Robinson et al., 2001; Bradley and Robinson, 2005). However, the use of these instruments may lead to order crossovers. We denote such order crossovers as *expected*

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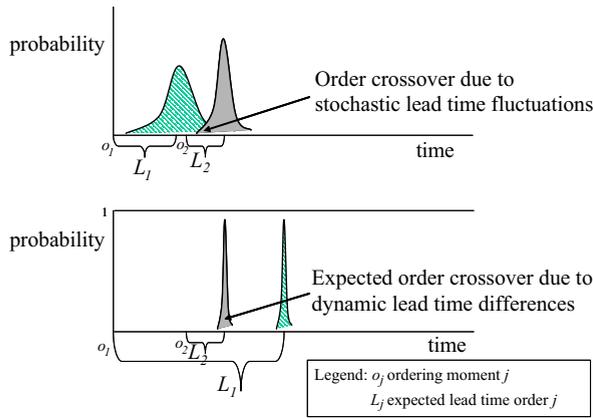


Fig. 1. Two types of order crossovers.

order crossovers, as they can be anticipated upon. Fig. 1 illustrates the difference between both types of order crossovers.

This paper studies the consequences of expected order crossovers for a single-item inventory system, with known (variable) ordering moments (discrete-time, periodic review, with variable review periods). It is assumed that the future ordering moments are known in advance, as well as the lead-times at these moments. Order crossovers may occur due to dynamic fluctuations in both lead-times and review periods. This paper assumes that the stochastic variation in these parameters is minor and can therefore be neglected. Demand is stochastic and forecasts for future demand are available and may be updated at the next ordering (i.e. decision) moment.

For this problem, Gaalman and Riezebos (2005) showed that the standard order-up-to policy that is available in inventory management systems leads to incorrect decisions in case of expected order crossovers. They derived an improved myopic order-up-to policy based on a stochastic dynamic programming approach.

In this paper, we show that the improved policy is still heuristic in nature, as it neglects several control options that are available on the various ordering moments and makes some restrictive assumptions with respect to the separability (i.e., decomposability) of the stochastic dynamic programming problem. We propose further improvements in the policy for situations where a quadratic cost function is appropriate.

## 2. Order crossovers and inventory ordering policies

The originating work on the optimal inventory equation defines an optimal ordering policy as a set

of functions that specify how much to order in each of the  $J$  stages and yield minimum total expected costs (Bellman et al., 1955). The decision in stage  $j$  is based on a system state variable. In most inventory models, the stock level is used as system state variable, but Aviv (2003) notes that, in general, the system state variable can be considered as a multi-dimensional vector, consisting of various variables that might have impact on the current decision.

The standard periodic review order-up-to policy with stochastic demand, known ordering moments and known lead-times is treated in all standard inventory modules of ERP systems. It uses the echelon inventory position at the moment of ordering as system state variable. Usually it is presented in textbooks (e.g. Silver et al., 1998; Tersine, 1988) with fixed review periods, but mathematically the formulation for variable review periods is similar:

$$Q_j = \hat{D}_{o_j, r_{j+1}}^{o_j} + M_{r_{j+1}} - E_{o_j}, \tag{1}$$

where

$$E_{o_j} = I_{o_j} + \sum_{t: o_t < o_j \wedge r_t \geq o_j} Q_t \tag{2}$$

and

$$E_{o_j} = E_{o_{j-1}} + Q_{j-1} - D_{o_{j-1}, o_j}^{act} \tag{3}$$

$Q_j$  is the size of the  $j$ th order that is issued, decided at order moment  $o_j$ ,  $L_j$  the lead-time of the  $j$ th order,  $M_t$  the minimum required stock just before time  $t$ ,  $O = \{o_j; j = 1, \dots, J\}$  the ordered set of ordering moments ( $o_j < o_{j+1}$ ),  $R = \{r_j; j = 1, \dots, J\}$  the set of arrival moments ( $r_j = o_j + L_j$ ),  $E_t$  the echelon inventory position at time  $t$ ,  $I_t$  the net on hand inventory at time  $t$ ,  $D_{t,u}^{act}$  the actual demand from time  $t$  to  $u$ ,  $\hat{D}_{t,u}^s$  at time  $s$  forecasted demand from time  $t$  to  $u$ .

By convention,  $E_t$  represents the situation just before time  $t$ ,  $\hat{D}_{t,u}^s = \hat{D}_{t,l}^s + \hat{D}_{l,u}^s$  if  $u \geq l$ , and  $\hat{D}_{t,u}^s = 0$  if  $u \leq t$ .

Formula (1) expresses the order size in terms of the difference between the total amount of items required before the next order that can be ordered arrives (at  $r_{j+1}$ ):  $\hat{D}_{o_j, r_{j+1}}^{o_j} + M_{r_{j+1}}$  and the currently known amount of items that will become available to fulfill these requirements:  $E_{o_j}$ . Hence it is an order-up-to policy (also known as a base stock policy), aiming at an end-of-period inventory equal to level  $M_{r_{j+1}}$ , which we denote as the safety stock level. This level can either be determined simultaneously with the order size or it can be set in

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