Int. J. Production Economics 122 (2009) 269–275

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Inventory model for an inventory system with time-varying demand rate

Michinori Sakaguchi

Faculty of Economic Sciences, Hiroshima Shudo University, 1-1, Ozuka-Higasi-Ichoume, Asaminami-ku, Hiroshima-shi 731-3195, Japan

ARTICLE INFO

Available online 21 June 2009

Keywords:
Inventory model
Probabilistic demand
Dynamic programming and economic order quantity

ABSTRACT

The standard inventory problems of the multi-period have been modeled under different situations. Specifically we have considered the demand subjects of a continuous distribution and a discrete distribution, and whether the demand of each period is unchanged or not. A method to get an economic order quantity in inventory systems with discrete and unchanged demand was presented in a previous paper, and this method has been generalized to an inventory model with varying continuous demand. However, it was not achieved due to there being many classified cases in the general situations. In this article the above method is discussed in the case discrete demand to determine whether it increases or decreases from period to period. A theoretical method is presented by using previous results and some examples are given which suggest how the concept can handle on inventory system. In order to make the decision, an algorithm is also presented under some conditions, and examples are shown by using the computer software program, Mathematica, which helps to explain the findings. In general cases, we view the optimal policy in the inventory problems in only a few periods.

1. Introduction

Probabilistic inventory models of the multi-period have been studied in which some conditions are searched to help obtain an optimal policy, providing that the total cost function of a single period is known extensively (Sakaguchi and Kodama, 2002). In those models, demand of each period is assumed to be unchanged for simplicity. These facts were applied to the case varying demand. However, it remains difficult to get the precise economic order quantity. Moreover, it was not achieved because there were many cases in the general situation. Therefore, the study initially researched the inventory model in the restricted case where demand decreased over time (Sakaguchi and Kodama, 2005). A model with exponential demand is researched in Sakaguchi (2007a) since it is easier when demand is subjected to an exponential distribution. In this paper we will continue to develop the method stated above where the case demand is discrete. The results in Sakaguchi and Kodama (to appear) inspire us to consider some examples in Sakaguchi (2007b) and subsequently generate a lot of demanding calculations.

The model investigated in this article is as follows. Let h and p be the holding cost and the shortage costs per unit per period, respectively, and let c be the purchasing cost per unit. Let us denote by z the amount on hand in initial period after a regular order is received and assume that demand in a single period be a discrete random variable. The decision criterion of single period is the minimization of the expected cost which includes the purchasing, holding and shortage costs. That is, the expectation $E\{C(b,z)\}$ of the total cost $C(b,z)$ is

$$E\{C(b,z)\} = c(\text{purchasing quantity}) + hE\{\text{holding quantity}\} + pE\{\text{shortage quantity}\}.$$

E-mail address: sakaguti@shudo-u.ac.jp

0925-5273/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2009.06.003

Our objective is to obtain a value z at which $E\{C(b,z)\}$ is minimized. Then, when the initial inventory quantity is x, the amount of the replenishment quantity is 0 or $z-x$.

The basic facts about our model are stated in Sakaguchi (2007a) and we refer the general notion of the inventory management to Silver et al. (1998).

Let N be the number of periods in the inventory model. In order to analyze the structure of our inventory models, many kinds of the functions are used in this article. The fundamental ones are $H^i(z)$ and $w^i(z)\,(z=1,2,\ldots,N)$ which are defined by the equations

$$E\{C(b_i,z)\} = -cx + H^i(z),$$

$$\Delta E\{C(b_i,z)\} = c - p + (h+p)w^i(z),$$

where b_i is a random variable of demand quantity in the ith period. Two conditions: D1 and D2 on the functions $H^i(z)$ are assumed to get the fundamental Theorem 4 and it leads us to an algorithm of seeking an optimal policy.

Two distributions, a uniform distribution and a Poisson distribution, are considered in this paper as making inventory examples. The statue of time-varying demand materializes by changing a parameter of distribution. We denote by DEC a decreasing demand from period to period and denote by INC an increasing one. Though we deal only with the case demand subjects to a discrete distribution, the model is built by using a continuous function $g(x)$ that indicates occurrence of a demand quantity along time in one period.

The functions

$$w_n^i(z) \quad (n=1,2,\ldots,N-1,\ i=1,2,\ldots,N-n+1)$$

are constructed inductively. Put

$$\kappa = \frac{p-c}{h+p}.$$

In the case of DEC, a method, used to obtain the economic order quantity from period i through period $i+n-1$ is achieved by solving an inequality

$$w_n^i(z) \ge \kappa + \frac{\alpha c}{h+p},$$

where α is a discount factor.

In the case of INC, it is too complicated and consequently we view only problems of a few periods. Finally, a lot of examples are shown that are computed by making use of the computer software Mathematica.

2. Inventory model

The inventory models in this paper are the dynamic inventory ones in which demands are varying period by period.

2.1. Notation and assumptions

Let us use notation as follows:

t	length of period
N	number of periods
c	purchasing cost per unit
h	holding cost per unit per period
p	shortage cost per unit per period with $c<p$
$\phi_i(b)$	probabilistic density function of demand quantity b at period $i\ (1\le i\le N)$
$g(x)$	function which has a continuous derivative on [0, 1] with $g(0)=0$, $g(1)=1$ and $g'(x)>0\ (0\le x\le 1)$
$G(y)$	$\int_0^y g(x)\,dx,\ y(0\le y\le 1)$
α	discount factor $(0<\alpha<1)$

$$\kappa = \frac{p-c}{h+p}$$

Suppose the following conditions hold:

- An order is instantaneously replaced without the fixed cost.
- Each replenishment is made at the beginning of each period. We often use a variable x which means the stock level before a regular order is taken at each period and a variable z presents the initial stock quantity after a regular order. Therefore a replenishment quantity is $z-x$.
- Demand quantity b of ith period is subject to a discrete distribution with a probabilistic density function $\phi_i(b)$ and they are independent of each other. Moreover if b is not an integer with $b\ge 0$, then $\phi_i(b)=0$.
- Demand of ith period occurs according to the function $g((T-it)/t)b$ at time $T(it\le T\le (i+1)t)$. That is, the inventory level at time T is $z-g((T-it)/t)b$.

2.2. Functions

In order to get the optimal policy, the following functions will be in use:

$f_n^i(x)$	the sum of the expected total cost from period i to period $i+n-1$ with the initial inventory level x at the time i before a regular order, provided an optimal policy is done at each opportunity
$I_1^i(b,z)$	the average inventory quantity at period i when the initial stock after a regular order is z and the amount of demand in period i is b
$I_2^i(b,z)$	the average shortage quantity at period i when the initial stock after a regular order is z and the amount of demand in period i is b
$C^i(b,z)$	the total cost of period i

The expectations of each function are given in Sakaguchi and Kodama (2005):

$$E\{I_1^i(b,z)\} = \begin{cases} 0 & \text{if } z\le 0,\\ \displaystyle\sum_{b=0}^{z}(z-G(1)b)\phi_i(b) & \\ \quad + \displaystyle\sum_{b=z+1}^{\infty}(zg^{-1}(z/b) & \\ \quad -bG(g^{-1}(z/b)))\phi_i(b) & \text{if } z\ge 1.\end{cases}$$

$$E\{I_2^i(b,z)\} = \begin{cases} G(1)m_i - z & \text{if } z\le 0,\\ \displaystyle\sum_{b=z+1}^{\infty}\{[G(1)-G(g^{-1}(z/b))]b & \\ \quad -z[1-g^{-1}(z/b)]\}\phi_i(b) & \text{if } z\ge 1,\end{cases}$$

where m_i is the mean of $\phi_i(b)$.

As was stated earlier in the introduction our criterion in this model of a single period is to minimize the expectation of total cost.