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A solution for the intractable inventory model when both demand and lead time are stochastic

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ABSTRACT

We consider the reorder point, order quantity inventory model where the demand, *D*, and the lead time, *L*, are independently and identically distributed (*iid*) random variables. This model is analytically intractable because of order crossover. However, we show how to resolve the intractability by empirical means, for example, by regression relationships produced by simulation and factorial experiments. Using a normal approximation, we show how to obtain regression equations for the optimal cost and the optimal policy parameters (here the order quantity and the safety stock factor) in terms of the problem parameters (ordering cost per order, holding cost per unit per unit time, shortage cost per unit, the standard deviation of demand, and the standard deviation of lead time).

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1. Introduction

We examine the general stochastic inventory model, where both the demand rate, *D*, and the lead time, *L* are independently and identically distributed (*iid*) random variables. This model has remained analytically intractable because of the problem of order crossover, which distorts the original distribution of the lead time and consequently that of the demand during the lead time.

1.1. Order crossover and its effects

Order crossover occurs when orders are not received in the same sequence in which they are placed, as for example the order that was placed second is delivered first. Let us illustrate by means of an example, where the lead time is gamma-distributed (the gamma is the conventional characterization of stochastic lead time), and suppose that we placed 100 orders at equal intervals in time, say at the beginning of each of the 100 periods. We display the interplay of these orders in Table 1, where we see that the order placed in period 4 comes in period 2, ahead of the orders placed in periods 2 and 3. We call that *order crossover*, whose effects are the following:

- The original lead time is now transformed to what we call *effective lead time* (*ELT*), which is the time between, say, placing the first order and getting the first delivery.

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Table 1
An example of order crossover with gamma lead times.

Time period	Lead time	Arrival time	Sorted arrival time	Effective lead time (ELT)
1	2.05	3.05	3.05	2.05
2	3.42	5.42	5.12	3.12
3	2.53	5.53	5.42	2.42
4	1.12	5.12	5.53	1.53
5	3.36	8.36	7.21	2.21
6	1.21	7.21	8.36	2.36
7	2.03	9.03	9.03	2.03
8	2.73	10.73	10.00	2.00
9	1.00	10.00	10.73	1.73
10	1.65	11.65	11.65	1.65
–	–	–	–	–
–	–	–	–	–
99	2.12	101.12	101.82	2.82
100	1.82	101.82	102.20	2.20

Descriptive statistics : *L*, *ELT*

Variable	N	Mean	StDev	Variance	CoefVar	Minimum
L	100	1.9610	0.8759	0.7672	44.67	0.4674
ELT	100	1.9610	0.6346	0.4027	32.36	0.5329

Variable	Median	Maximum	Skewness	Kurtosis
L	1.8850	4.9912	0.79	0.71
ELT	1.9948	3.9887	0.31	0.25

- The mean of the *ELT* is the same as that of the original lead time, but its variance is usually much less.
- Now we should use the demand during the effective lead time (*DDELT*) instead of the conventional lead time demand (*LTD*).
- The *ELT* is an *AR*(1) process.

1.2. The literature

For recent articles on order crossover, see Robinson et al. (2001), Bradley and Robinson (2005), Robinson and Bradley (2008) and Hayya et al. (2008). However in prior literature, when, both D and L were *iid random variables*, a solution was arrived at by obtaining the compound distribution of demand during lead time and then using a normal approximation to calculate the 'optimal cost' and the 'optimal policy parameters,' such as the optimal order quantity, Q^*, and the optimal safety stock factor, z_o^*. But, unfortunately, these compound distributions (for example, Burgin, 1972; Ord and Bagchi, 1983) are specious, because they neglect order crossover. We note at the outset that we are taking literary license in the use of word 'optimal' in the present paper, as the equations we shall use are the result of a heuristic approximation (Hadley and Whitin, 1963, Chapter 4).

In our development, we show through factorial experiments that the 'optimal cost', C^*, the 'optimal order quantity', Q^*, and the 'optimal safety stock factor', z_o^*, can be written as regression functions of the problem parameters (the ordering cost, the holding cost, the shortage cost, mean and standard deviation of the demand rate, and mean and standard deviation of the lead time). We do that using three different distributions for lead time: the normal, the Poisson, and the Gamma.

In this paper, we attempt to answer this question:

In case of order crossover with both D and L iid random variables, can we circumvent the analytical complexity and produce an optimal cost, optimal order quantity, and an optimal reorder point through the use of regression equations?

1.3. Organization of the paper

In Section 2, we tackle the order quantity, reorder point inventory model, where the shortage penalty is per unit short. This is carried forward in Section 3, where we use the normal approximation and where we perform factorial experiments, using a simulation algorithm that is given in Appendix A. In Section 4, we use a numerical example to display the power of our regression procedure. Section 5 summarizes, and in Appendix B, we provide a list of symbols and acronyms used.

2. The cost per unit short model

We choose the cost (B_2) per unit short model (Silver et al., 1998, p. 263), because it is the simplest and thus the most popular. But please note the qualification below with respect to the use of B_2. In our construction, the inventory cost is

$$C(Q, X, z_o) = AE(D)/Q + (Q/2 + z_o\sigma_x)h + \frac{DB_2}{Q}\sigma_x G(z_o), \tag{2.1}$$

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