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An integrated production-inventory system in a multi-stage multi-firm supply chain

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ABSTRACT

We first generalize a number of integrated models with/without lot streaming and with/without complete backorders under the integer–multiplier coordination mechanism, and then individually derive the optimal solution to the three- and four-stage model, using algebraic methods of complete squares and perfect squares. We subsequently deduce optimal expressions for some well-known models. For our model, we check that the optimal solution, which is algebraically derived, is a global one. We present three numerical examples for illustrative purposes. We finally suggest some future research work involving extension or modification of the generalized model.

1. Introduction

Increasing attention has been given to management of multi-stage multi-firm supply chains in recent years. This is due to rising competition, shorter life cycles of products, quick changes in today's business environment and severity of green issues. Integrated deteriorating production-inventory models incorporating the factor of environmental consciousness can be found in Yu et al. (2008), Chung and Wee (2008), and Wee and Chung (2009). Integration of a (green) supply chain is now crucial to successful international business operations since an integrated approach improves global systems' performance and cost effectiveness. Besides integrating operations of all members in a supply chain, improvement of the traditional method of solving inventory problems is also necessary. Without using derivatives, Grubbström (1995) first derived optimal expressions for the classical EOQ (economic order quantity) model using the unity decomposition method, which is an algebraic approach. Adopting this method, Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) derived optimal expressions for EOQ and EPQ (economic production quantity) models with complete backorders. In this paper, a generalized model for a three- or four-stage multi-firm integrated production-inventory system is solved, using the methods of complete squares and perfect squares proposed in Leung (2008a,b), which are simple algebraic methods; ordinary readers unfamiliar with differential calculus can also easily understand how to derive optimal expressions of decision variables and the objective function.

2. Assumptions, symbols and designations

The integrated production-inventory model is developed under the following assumptions:

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(1) A single item is considered.
(2) There are two or more stages.
(3) Production and demand rates (with the former greater than the latter) are independent of production or order quantity, and are constant.
(4) Unit cost is independent of quantity purchased, and order quantity will not vary from one cycle to another.
(5) Setup or ordering costs are different for all firms in the chain.
(6) Holding costs of raw materials are different from those of finished products.
(7) Holding costs of raw materials are different for all firms in the chain.
(8) Holding costs of finished goods are different for all firms in the chain.
(9) Shortages are allowed for some/all retailers and are completely backordered, and all backorders are made up at the beginning of the next order cycle.
(10) Neither a wait-in-process unit, nor a defective-in-transit unit, is considered.
(11) Each upstream firm implements perfect inspection to guarantee that defective units are not delivered to any retailer.
(12) All firms have complete information of each other.
(13) The number of shipments of each supplier, manufacturer, assembler or retailer is a positive integer.
(14) The planning horizon is infinite.

The following symbols (some as defined in Leung, 2009) are used in expression of the joint total relevant cost per year.

D_{ij} = demand rate of firm j (=1, ..., J_i) in stage i (=1, ..., n) (units per year)
P_{ij} = production rate of firm j (=1, ..., J_i) in stage i (=1, ..., $n-1$) (units per year)
S_{ij} = setup or ordering cost of firm j (=1, ..., J_i) in stage i (=1, ..., n) ($ per cycle)
g_{ij} = holding cost of incoming raw materials of firm j (=1, ..., J_i) in stage i (=1, ..., $n-1$) ($ per unit per year)
h_{ij} = holding cost of finished goods of firm j (=1, ..., J_i) in stage i (=1, ..., n) ($ per unit per year)
$b_{nj} \equiv b_j$ = backordering cost of finished goods of firm j (=1, ..., J_n) in stage n ($ per unit per year)

Without lot streaming for any firm in stage i (=1, ..., $n-1$), we denote

$t_{nj} \equiv t_j$ = backordering time of firm j (=1, ..., J_n) in stage n (t_j are decision variables, each with *non-negative* real values) (a fraction of a year)
$T_{nj}^{(b)} \equiv T$ = basic cycle time of firm j (=1, ..., J_n) in stage n (T is a decision variable with *non-negative* real values) (a fraction of a year)
$T_{ij} = T\prod_{k=i}^{n-1} K_k$= integer–multiplier cycle time of firm j (=1, ..., J_i) in stage i (=1, ..., $n-1$) ($K_1, \dots, K_{n-1}$ are decision variables, each with *positive* integral values) (a fraction of a year)
$TC_{ij}^{(0)}$= total relevant cost of firm j (=1, ..., J_i) in stage i (=1, ..., n) ($ per year)
$JTC(K_1, \dots, K_{n-1}, T, t_j) = \sum_{i=1}^{n}\sum_{j=1}^{J_i} TC_{ij}^{(0)}$ = joint total relevant cost as a function of $K_1, \dots, K_{n-1},\ T$ and t_j (the objective function) ($ per year)

With lot streaming for any firm in stage i (=1, ..., $n-1$), we denote

$u_{nj} \equiv u_j$ = backordering time of firm j (=1, ..., J_n) in stage n (u_j are decision variables, each with *non-negative* real values) (a fraction of a year)
$U_{nj}^{(b)} \equiv U$ = basic cycle time of firm j (=1, ..., J_n) in stage n (U is a decision variable with *non-negative* real values) (a fraction of a year)
$U_{ij} = U\prod_{k=i}^{n-1} L_k$ = integer–multiplier cycle time of firm j (=1, ..., J_i) in stage i (=1, ..., $n-1$) ($L_1, \dots, L_{n-1}$ are decision variables, each with *positive* integral values) (a fraction of a year)
$TC_{ij}^{(1)}$ = total relevant cost of firm j (=1, ..., J_i) in stage i (=1, ..., n) ($ per year)
$JTC(L_1, \dots, L_{n-1}, U, u_j) = \sum_{i=1}^{n}\sum_{j=1}^{J_i} TC_{ij}^{(1)}$ = joint total relevant cost as a function of $L_1, \dots, L_{n-1},\ U$ and u_j (the objective function) ($ per year)

To simplify the presentation of the subsequent mathematical expressions, we designate

$$\varphi_{ij} = \frac{D_{ij}}{P_{ij}} \quad \text{and} \quad \bar{\varphi}_{ij} = 1 - \varphi_{ij} \quad \text{for } i = 1, \dots, n-1;\ j = 1, \dots, J_i, \tag{1}$$

$$S_{iJ} = \sum_{j=1}^{J_i} S_{ij} \quad \text{for } i = 1, \dots, n, \tag{2}$$

$$H_1 = \sum_{j=1}^{J_1} D_{1j}[\varphi_{1j}(g_{1j} + h_{1j}) + h_{1j}], \tag{3}$$