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A two-echelon inventory/distribution system with power demand pattern and backorders

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A R T I C L E I N F O

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A B S T R A C T

This paper addresses the implications of considering power demand pattern and backorders in the one-warehouse *N*-retailer problem. Specifically, we assume that items at the retailers are withdrawn from the inventory following a power pattern. The objective function consists of the sum of ordering, holding and backordering costs at all installations. This objective function depends on the class of inventory policies that we choose for the formulation of the problem. In general, obtaining the cost function is an arduous task since the average inventory at the warehouse could be difficult to determine. However, if we apply single-cycle policies the computation of the total average cost becomes possible. Under these assumptions we show that the one-warehouse *N*-retailer system with power demand pattern and backorders can be formulated as a mixed non linear programming problem.

1. Introduction

In practice, inventory systems usually involve a number of installations which are coupled to each other. For instance, one central warehouse can supply goods to a chain of stores. This class of inventory systems arises in both distribution and production context. In case of a distribution system, products are delivered to a set of installations located over large geographical areas. Accordingly, local stock points should be established close to the customers to guarantee a satisfactory service level. These local sites replenish their stocks from a central warehouse close to the production facility. In the production framework, stocks of raw materials, components and finished products are coupled to each other in a similar way. It is worth noting that in these situations decisions made by a member of a chain can affect to the rest of locations. Hence, it is necessary that all members of the supply chain collaborate and integrate their order policies to achieve a more efficient inventory control.

This paper specifically focuses on an inventory/distribution system consisting of one central warehouse, which supplies products to a set of *N* retailers that in turn cover customer demand. The one-warehouse *N*-retailer problem has been extensively analyzed in the literature by authors such as Schwarz (1973), Graves and Schwarz (1977), Williams (1981, 1983), Roundy (1985), Muckstadt and Roundy (1993) and Abdul-Jalbar et al. (2003, 2005, 2006), among others. For an introduction to this problem the reader is referred to Silver et al. (1998), Zipkin (2000) and Axsäter (2000). In most cases, the problem assumes that customer demand at the retailers occurs at constant rate and is uniformly distributed through the planning horizon. Additionally, it is assumed that demand should be satisfied without backorders. Nevertheless, Mitchell (1987), Atkins and Sun (1995), Sun and Atkins (1997) and Chen (1998, 1999, 2000) extended the model to allow shortages.

However, assuming that customer demand at each retailer is uniformly distributed through the planning horizon is an idealistic assumption which is not always

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appropriate or applicable. In real life situations the demand of an item toward the beginning of a period, e.g., a week, can be smaller or greater than the demand at the end of the period. The goal of this paper is to analyze the one-warehouse N-retailer problem under this situation, that is, assuming that customer demand at each retailer follows a power pattern. Moreover, we also admit that unmet demand in one cycle can be satisfied by orders in subsequent periods, i.e., we allow backorders. Thus, we should compute a joint cost function integrating the ordering, holding and backordering costs at each location. Unfortunately, obtaining this expression is, in general, a quite complex task since the computation of the average inventory at the warehouse is not straightforward. Hence we focus our attention on a special class of ordering policies, namely single-cycle policies, which simplifies the formulation of the problem. In particular, we show that after some rearranging and calculations the formulation of the one-warehouse N-retailer problem with power demand pattern and backorders yields the formulation of the basic one-warehouse N-retailer problem. In addition, it is also shown that the expression of the total average cost generalizes the one obtained for the single-level case.

In the next section we introduce the notation required to state the problem. In Section 3 the problem is formulated as a mixed non linear programming problem by using single-cycle policies. In addition, we show that all methods that have been proposed in the literature to solve the one-warehouse N-retailer problem with constant demand and without backorders can be easily modified to solve the problem addressed in this paper. Finally, in Section 4, concluding remarks are reported and future research is proposed.

2. Notation and preliminaries

We assume that customer demand at the retailers is known and follows a power pattern through each cycle. This latter assumption allows to model real-world instances in which the demand is not uniformly distributed through the time but it is mainly concentrated in a time interval within the cycle. Besides, we assume that items are instantaneously included to the inventory after an order is placed, and demand in one period can be met by orders in subsequent cycles. For each installation we consider ordering and holding costs and, at the retailers, we also admit backordering costs.

The problem involves the following parameters for each retailer $j = 1, \ldots, N$: demand d_j through the ordering cycle at retailer j, positive index of the power demand pattern p_j, inventory quantity S_j at the beginning of a period, fixed ordering cost k_j, holding unit cost h_j and backordering unit cost b_j. Additionally, we denote by k_0 and h_0, respectively, the ordering and holding unit costs at the warehouse. The decision variables are t_j and t_0, which represent, respectively, the time interval between consecutive replenishments at retailer $j = 1, \ldots, N$, and at the warehouse. Finally, C_j, C_0 and C_T denote, respectively, the average cost at retailer j, the average cost at the warehouse and the average overall cost of the system.

Furthermore, since customer demand at each retailer follows a power pattern, the inventory quantity at retailer j at a time instant t is defined by

$$Q_j(t) = S_j - d_j t_j \left(\frac{t}{t_j}\right)^{1/p_j}$$

Notice that if $p_j = 1$, then $Q_j(t) = S_j - d_j t$, which corresponds to the constant demand case analyzed by Mitchell (1987), Atkins and Sun (1995), Sun and Atkins (1997) and Chen (1998, 1999, 2000), among others. On the other hand, if $p_j < 1$, then Q_j is a decreasing concave function and the demand is more concentrated at the end of the period. In contrast, if $p_j > 1$, then Q_j is a decreasing convex function and the demand is more concentrated at the beginning of the period. In the latter case, the behavior of the inventory level is similar to the one obtaining when linear stock dependent demand rate or deteriorating items is assumed. The problem under these assumptions has been extensively studied in the literature. For example, Baker and Urban (1988), Datta and Pal (1990) and Zhou (2002) have analyzed the stock dependent demand rate case. Yang and Wee (2000, 2002), Wee et al. (2006) and Zhou and Wang (2007), among others, have addressed the problem assuming deteriorating or perishable items. Some authors have considered both assumptions, see for example Giri et al. (1996), Padmanabhan and Vrat (1995), Giri and Chaudhuri (1998) and Balkhi and Benkherouf (2004).

3. Statement of the problem

To the best of our knowledge, we do not know of any previous work dealing with the one-warehouse N-retailer problem with power demand pattern and backorders. Many references in the specialized literature analyze the problem assuming that customer demand at the retailers occurs at constant rate and backorders are not allowed. Roundy (1985) showed that the structure of the optimal policies is quite complex even when these idealistic assumptions are considered, and hence they are difficult to implement in practice. Accordingly, most authors have focused on studying sets of policies which are both effective and easy to compute. In particular, single-cycle policies are within this category since they are stationary and nested. We say that a policy is stationary when both the order quantity and the time interval between two consecutive orders are constant. Additionally, a policy is said to be nested when an order placed at the warehouse makes the retailers to place an order as well. As a result, there should be integers $n_1, n_2, \ldots, n_N$ such that $n_1 t_1 = n_2 t_2 = \cdots = n_N t_N = t_0$, where n_j denotes the number of replenishments at retailer j during a cycle t_0, $j = 1, \ldots, N$. This class of policies has been used by Schwarz (1973), Graves and Schwarz (1977), Muckstadt and Roundy (1993) and Abdul-Jalbar et al. (2003, 2006) among others authors, to formulate diverse inventory/distribution problems.

Throughout this paper we use single-cycle policies to determine the average overall cost of a one-warehouse N-retailer system with power demand pattern and